

MATH 5 — PREREQUISITES FOR MATH 6

1. PROGRAM

- Algebraic expressions. Commutativity, associativity, distributivity.
- Equations. Solving word problems with equations.
- Powers of 2.
- Binary numbers.
- Powers. Negative powers. Scientific notation.
- $a^2 - b^2 = (a - b)(a + b)$
- Square roots.
- Pythagorean theorem.
- Basic probability theory: addition rule, complement rule, product rule.
- Choosing with and without repetition. Permutations.
- Geometry: parallel lines and angles (alternate interior, alternate exterior, corresponding).
- Parallelogram, various definitions, properties.
- Congruence tests for triangles (SAS, ASA, SSS).
- Isosceles triangle. Median, bisector, height.
- Trapezoid. Its midline. Area.

2. PROBLEMS

1. Rewrite each of the expressions below in the simplest possible form, by collecting the like terms if possible.

$$\begin{array}{lll} 2x + 7 + 5x + 2 + 3x & 3x + 9 + 5xy + 2xy + 3 & 3(2x - 1) + x \\ 2a(a - 2) - a(a - 1) & (2x - 1)(x + 1) & \end{array}$$

2. An apple cost 9 cents, and an orange 15 cents. Elena bought some apples and oranges, 20 fruit in all, and paid \$2.64. How many apples and how many oranges did she buy?
3. A boy had a bag of apples. He gave $\frac{1}{2}$ of them to his parents, $\frac{1}{5}$ to his brother, $\frac{1}{4}$ to his sister and the last apple he ate himself. How many apples did he originally have?
4. Simplify the following expressions

$$(a) x + 4(1 - x) \quad (b) 2 + 5x - 4(3 - x) \quad (c) 5(x - 1) - 3(2x + 1)$$

5. If you take half my age and add 7, you get my age 13 years ago. How old am I?
6. Two secretaries, Barbara and Mary, need to type a 100 page document. Barbara can type it in 4 hours; Mary types slower, so it would take her 5 hours to do this. How

fast can they type it together if they divide the work between two of them in the most efficient way?

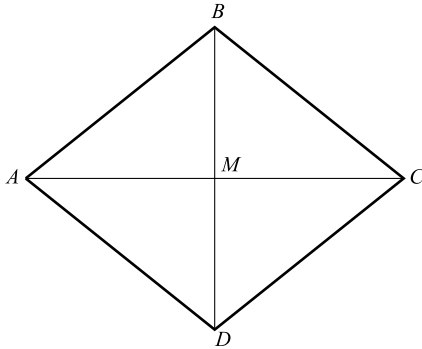
7. Find the sum $1 + 2 + 4 + \cdots + 2^n$ (the answer, of course, will depend on n). [Hint: first try computing it for several small values of n : find $1 + 2$, then $1 + 2 + 4$, then $1 + 2 + 4 + 8$. See if you can notice a pattern. After this, try formulating a general rule.]
8. Convert the decimal numbers to binary:
9, 12, 24, 38, 45
9. Convert the following binary numbers to decimal:
101, 1001, 10110, 11011, 10101
10. Compute $110101_b + 111011_b$ without converting numbers to decimal form.
11. Simplify the following expression:

$$\frac{(x^2y^2) \cdot x^3}{x^2y^5}$$

12. Let $a = 2 \cdot 10^8$, $b = 10^5$. Compute $a^2 \cdot b$, $\frac{a}{b}$, $a^2 \div b^3$.
13. If $a = 2^{-13}3^9$, $b = 2^{11}3^{-7}$, what is the value of ab ? of a/b ?
14. Write the following numbers using scientific notation.
 - (a) the distance from Earth to Pluto is $\approx 7,527,000,000$ km;
 - (b) the distance from Earth to the star Sirius is $\approx 81,900,000,000,000$ km;
15. Factor the following number into primes: $99^2 - 9^2$. [Hint: you do not have to compute this number.]
16. Find the following square roots. If you can not find the number exactly, at least say between which two whole numbers the answer is, e.g., between 5 and 6.
 - (a) $\sqrt{81}$
 - (b) $\sqrt{10,000}$
 - (c) $\sqrt{10^8}$
17. If, in a right triangle, one leg has length 1 and the hypotenuse has length 2, what is the other leg?
18. Simplify: $(\sqrt{17})^2$, $(\sqrt{13})^4$, $(\sqrt{11})^3$, $\sqrt{2^4 3^6}$, $\sqrt{2^4 3^5}$.
19. We roll two dice. What is the probability of getting sum of two numbers equal to 4?
20. If we toss a coin 5 times, what is the probability that **at least one** will be heads?
21. A license plate consists of 3 letters, followed by three digits. How many possible license plates are there?
22. In one kind of lottery, they put balls with numbers 1 through 100 in a bag and then draw six balls at random (drawn ball is put aside and not returned to the bag). To win the lottery, one needs to guess all six numbers in correct order. What is the probability of this?

23. How many ways are there to seat 15 students in a classroom which has 15 chairs? If the room has 25 chairs?
24. In a meeting of 25 people, each person must shake hands with each other. How many handshakes will there be altogether?
25. At a certain meeting of 25 people, they decide to select a committee, which would have a chairman and 2 members. How many ways are there to do it?
26. Show that in a parallelogram, diagonally opposite angles are equal $\angle A = \angle C$, $\angle B = \angle D$
27. Let $ABCD$ be a quadrilateral such that $AB = BC = CD = AD$ (such a quadrilateral is called rhombus). Let M be the intersection point of AC and BD .

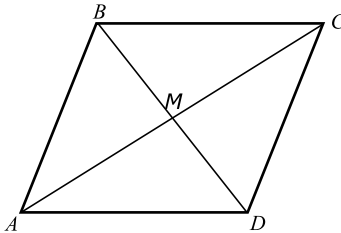
- (a) Show that $\triangle ABC \cong \triangle ADC$
 (b) Show that $\triangle AMB \cong \triangle AMD$
 (c) Show that the diagonals are perpendicular and that the point M is the midpoint of each of the diagonals.



[Hint: after doing each part, mark on the figure all the information you have found — which angles are equal, which line segments are equal, etc: you may need this information for the following parts.]

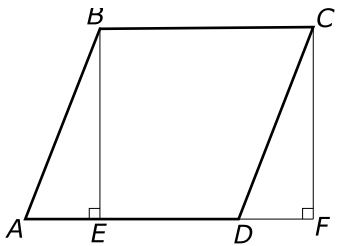
28. Let $ABCD$ be a quadrilateral such that sides AB and CD are parallel and equal (but we do not know whether sides AC and BD are parallel).

- (a) Show that triangles $\triangle AMB$ and $\triangle CMD$ are congruent.
 (b) Show that sides AC and BD are indeed parallel and therefore $ABCD$ is a parallelogram.

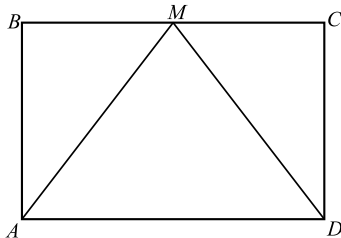


29. Let $ABCD$ be a parallelogram, and let BE, CF be perpendiculars from B, C to the line AD .

- (a) Show that triangles $\triangle ABE$ and $\triangle DCF$ are congruent.
 (b) Show that the area of parallelogram is equal to height \times base, i.e. $BE \times AD$.



30. In the figure to the right, $ABCD$ is a rectangle, and M is the midpoint of BC . Prove that then triangle AMD is isosceles.



31. (a) Show that if in a quadrilateral $ABCD$ diagonals bisect each other (i.e., intersection point is the midpoint of each of the diagonals), then $ABCD$ is a parallelogram. [Hint: find some congruent triangles in the figure.]
- (b) Show that if in a quadrilateral $ABCD$ diagonals bisect each other and are perpendicular, then it is a rhombus.

32. Find all lengths, angles, and area in the figure shown to the right.

