WORK DONE BY GAS.

APRIL 24, 2022

Theory recap

Last time recap. Last time we discussed some ideal gas processes and their graphical representation. All of that discussion relied on ideal gas equation of state:

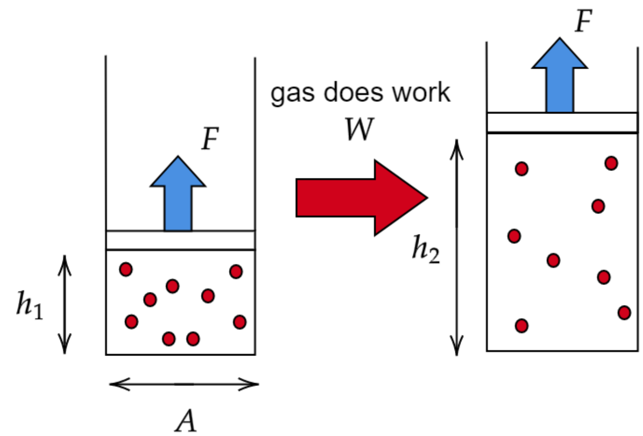
$$pV = nRT.$$

Work done by gas. Today we are considering a connection between mechanics and thermodynamics. We will learn how heat can be transferred to work using a gas.

First, what is work? As you recall from mechanics, work is force times displacement. So in order to have work, we need to have force and displacement.

Consider our favorite setup: some amount of gas in a container under a piston. If cross section area of the container (and the piston) is A and gas pressure is p, the force with which gas acts on the piston follows from definition of pressure:

$$p = \frac{F}{A} \implies F = pA.$$



Assume that the gas is expanding, so the piston moves up by height $h = h_2 - h_1$ (see figure) above its initial position. Work done by the gas on the piston in this case is

$$W = Fh = pA(h_2 - h_1) = p(Ah_2 - Ah_1) = p(V_2 - V_1) = p\Delta V.$$

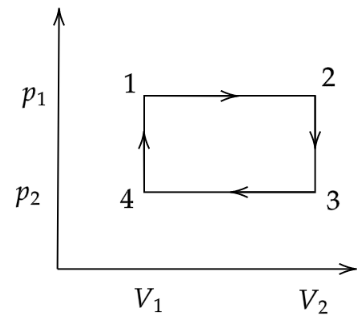
We used the fact that product of cross section area and height is volume of the gas (final volume is denoted V_2 and initial volume V_1) and denoted change of volume by ΔV. To sum up, we have derived the formula for work done by gas: work is done whenever volume is changed and for processes at constant pressure it is equal to pressure times change in volume:

$$W = p\Delta V.$$

Remember that work can be positive or negative: it is positive when force and displacement point in the same direction and negative when they point in the opposite directions. Let us consider two possible cases to show that our formula for work of the gas actually gets this right. When volume changes, it either increases or decreases. When volume increases, force on the piston and displacement of the piston both point up in our figure above. Therefore work should be positive - and from our formula $W = p\Delta V > 0$ because $\Delta V > 0$ (and p is always positive). On the other hand when volume decreases, displacement is in the opposite direction (down on our figure) than the force (which still points up). Therefore work should be negative and from our formula $W = p\Delta V < 0$ because $\Delta V < 0$.

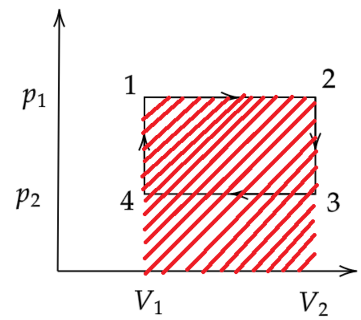
Work and the graphical representation. There is a nice relation between work of gas and graphical representation of processes which we discussed previously.

Let us consider a cyclic process $1-2-3-4-1$ consisting of two isobaric processes (1-2 and 3-4) and two isochoric processes (2-3 and 4-1), see figure to the right. What is the total work done by the gas during the cycle? The total work is equal to the sum of individual works done during every of the four parts of the cycle:



$$W_{total} = W_{12} + W_{23} + W_{34} + W_{41}$$

Let us look at these terms individually. Start with W_{12}. The process 1-2 is isobaric, meaning that the pressure is constant, so we could use our formula for work of the gas directly:



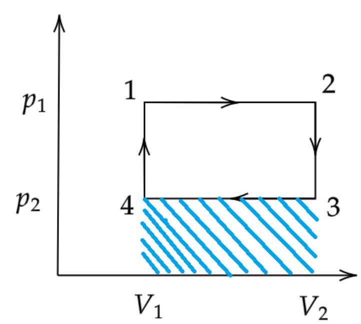
$$W_{12} = p_1(V_2 - V_1)$$

where notations are explained on the figure. We find that the work in process $1-2$ is numerically equal to area under the line 1-2 (which is the area of a rectangle), shown in red on the figure.

Now we need to find W_{23}. The process 2-3 is isochoric, which means that the volume does not change. Therefore the piston does not move and no work is done:

$$W_{23} = 0.$$

The process 3-4 is again isobaric but volume is decreasing, therefore the work turns out to be negative:



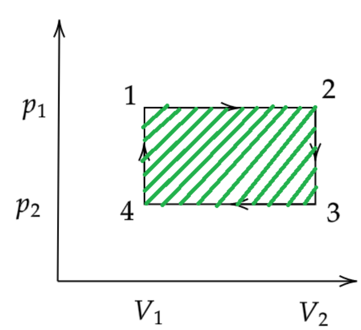
$$W_{34} = p_2(V_1 - V_2) < 0.$$

It is numerically equal to the area of the rectangle under line 3-4, shown in blue on the figure, taken with a minus sign.

Finally, the process 4-1 is again isochoric and no work is done, so

$$W_{41} = 0.$$

Summing all the terms up, we have



$$W_{total} = p_1(V_2 - V_1) + p_2(V_1 - V_2) = (p_1 - p_2)(V_2 - V_1)$$

We see that the total work is numerically equal to the area of rectangle which represents our cycle in $p-V$ coordinates, shown in green on the figure.

Homework

1. There is a cylinder with a piston. The mass of the piston is 100 kg, its area is 10 cm^2. The cylinder contains 28 g of nitrogen at $T_1 = 273$ K. The cylinder is heated up to $T_2 = 373$ K. How does the piston position change? How does the potential energy of the piston change? What work is done by the gas? Neglect atmospheric pressure.

2. How much hydrogen H_2 (in grams) is in a cylinder with a piston if it performs work of 400 J being heated from 250 K to 680 K? The gas pressure was maintained constant.

***3.** We did not have time to discuss this in class, but it is true for every cycle that work during the cycle is numerically equal to area of the region enclosed by the cycle (in the class we have only proven it for a "rectangular" cycle consisting of two isobaric and two isochoric processes). Using this fact, find the total work done by the gas during the cycle 1-2-3-1 shown on the figure.