Homework 24

Snell's law

Last class we discussed refraction. The refraction is the change of a light beam direction as it obliquely enters into one media from another media, say, from air to water. Similar to the reflection, the refraction can change the direction of light propagation. We can derive the law of refraction using Huygens-Fresnel principle. Let us consider light rays hitting the boundary between two translucent media. The upper is characterized by refractive index n_i, the lower one – by refractive index n_t.

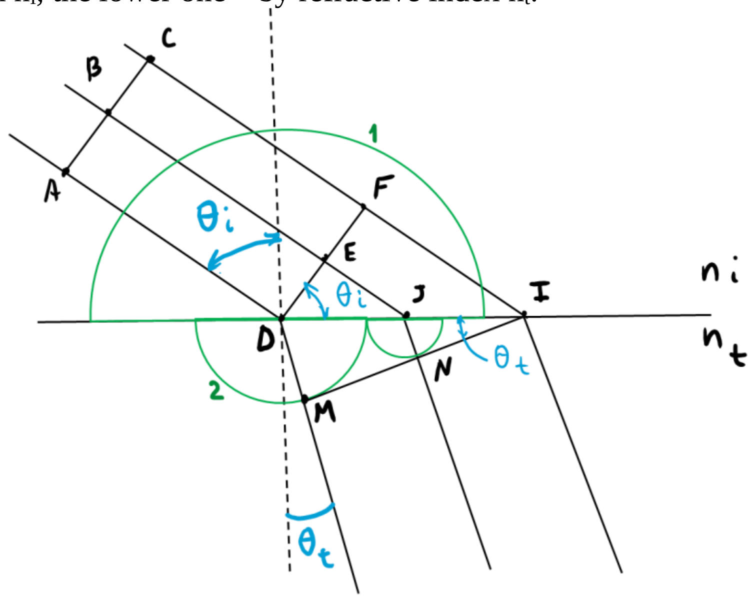


Figure 1.

We learned that in any material light propagates slower than in vacuum. The number which shows how many times the speed of light in a material is less than in vacuum is called the *index of refraction*, or *refractive index*. For example, the index of refraction of diamond is ~2.4. It means that light propagates in diamond 2.4 times slower than in vacuum.

The material with higher refractive index we will call "more optically dense". So in Figure 1light comes from less optically dense material to a material with higher optical density. Part of the light is reflected from the boundary. In the lower material the light propagates more slowly, so the radius of the "transmitted wavelet" from point D (green semi-circle 2) is less than this of the "reflected wavelet" from the same point (green semi-circle 1). The reflected rays are not shown to "unload" the figure. Lines AC, DF and MI represent the wavefronts, they are perpendicular to the rays.

The angle of incidence $\theta_i = \angle FDI$; the angle of refraction$\theta_t = \angle DIM$. Then, $sin\theta_i = |FI|/|DI|$ and $sin\theta_t = |DM|/|DI|$. From these we have:

$$\frac{sin\theta_i}{sin\theta_t} = \frac{|FI|}{|DI|} \cdot \frac{|DI|}{|DM|} = \frac{|FI|}{|DM|} \qquad (1)$$

Now let us assume that it takes time Δt for the light to travel from point F to point I. Then

$$|FI| = \Delta t \cdot v_i = \Delta t \cdot \frac{c}{n_i} \qquad (2)$$

$$|DM| = \Delta t \cdot v_t = \Delta t \cdot \frac{c}{n_t} \qquad (3)$$

, where v_i is the speed of light in the upper material, v_t is the speed of light in the lower material and c is the speed of light in vacuum. Plugging (2) and (3) into (1) we have

$$\frac{sin\theta_i}{sin\theta_t} = \frac{n_t}{n_i} \qquad (4)$$

Equation (4) is the Snell's law.
It is easy to obtain the expression for the sin$\boldsymbol{\alpha_r}$:

$$sin\theta_t = \frac{n_i}{n_t} sin\theta_i \qquad (5)$$

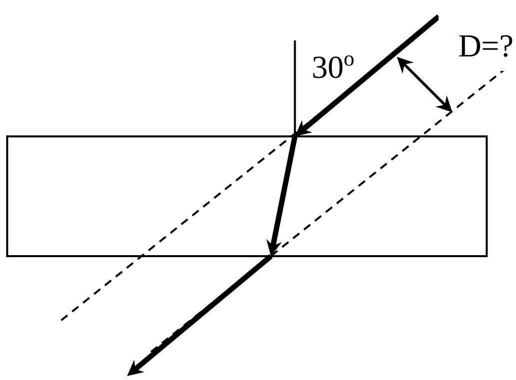
If you know the angle of incidence you can easily calculate the angle of refraction.
If the light moves from material with higher refractive index to material with lower refractive index then the angle of refraction is higher than the angle of incidence. In this case, at a certain angle of incidence the angle of refraction reaches 90^o.If we further increase the angle of incidence then no light will penetrate into the material with lower refractive index – all the light will be reflected from the interface. This phenomenon is called "***total internal reflection***", and the corresponding angle – "angle of total internal reflection" - $\boldsymbol{\alpha_t}$. It can be found using expression (5): we have to make α_r=90^o and replace α_i to α_t.

$$\alpha_{tot} = arc\ sin \frac{n_t}{n_i} \qquad (2).$$

If the light moves from low refractive index media to high refractive index one, n_r/n_i>1 and no $\boldsymbol{\alpha_t}$ exists.

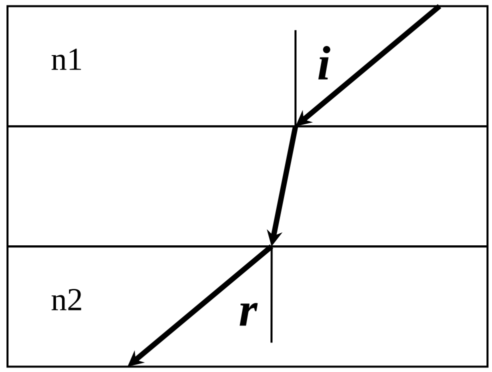
Problems:

1. A beam of light comes from air (the index of refraction n=1) and passes through a glass plate (the index of refraction n=1.5). The thickness of the plate is 1cm. The angle of incidence is 30^o. Find the displacement of the beam (i.e. the distance between beam axis before and after it passes through the plate – see picture below).

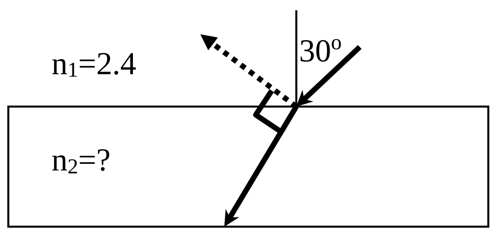


2. There are two transparent plates and liquid between them (see picture below). The indexes of refraction of the plates are $\boldsymbol{n_1}$ and $\boldsymbol{n_2}$. The beam of light propagates from

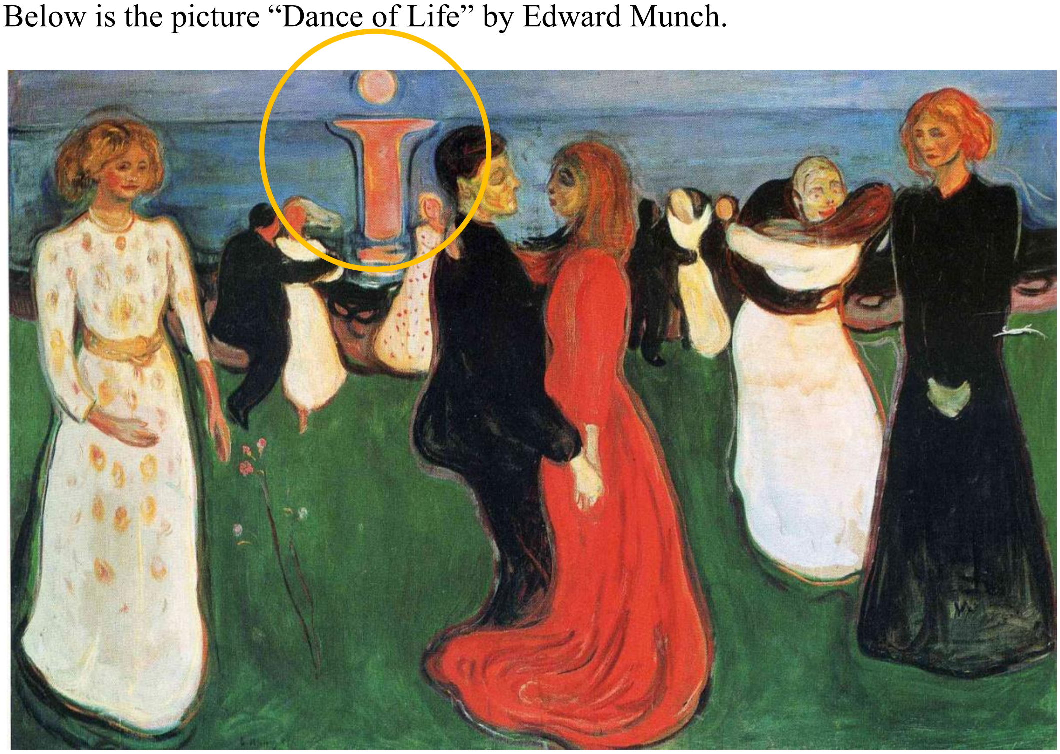
the upper plate at the angle ***i*** and enters the lower plate at the angle ***r***. Prove that $\sin(i)/\sin(r) = n_2/n_1$ independently on the thickness of the layer of liquid.



3. A beam of light hits the plane separating two transparent materials at the angle of 30^o (see picture below). The index of refraction of the upper material is $n_1=2.4$. Find the index of refraction of the lower material if the reflected beam is perpendicular to the refracted beam.



4. Below is the picture "Dance of Life" by Edward Munch.



In the upper part of the picture you can see a bright “light path” (inside the yellow circle) which “goes” on the surface of the water from the low sun. Try to explain this effect