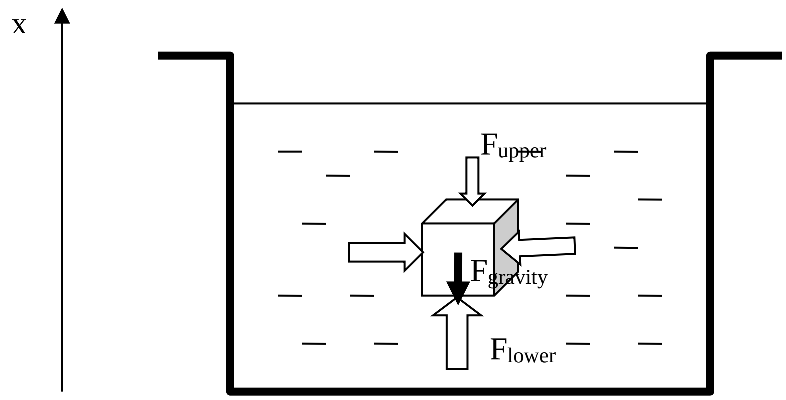
Homework 29

Buoyancy.

This time we will discuss pressure in liquids and buoyancy.

Any object immersed in liquid “feels” pressure. This pressure “tries” to squeeze the object. The physical reason for this pressure is gravity. (Surface tension can contribute to the pressure inside small droplets but we will not discuss it today). Intuitively we can understand that the deeper we plunge the higher weigh of the liquid we feel, so the higher is the pressure.

Let us see what the pressure in liquids depends on and how it can be calculated. Let us consider a glass of water. If we do not move the glass, the water inside is in equilibrium and does not move. This applies to any “part” of the water in the glass.



Let us consider a small cube of water inside the glass. The walls of this cube are imaginary, but the water inside is real. Since the cube does not move (the water does not flow inside the glass) the total net force applied to the cube is 0. We consider only the forces applied in vertical direction. There is a gravity force applied to the cube. It is:

$$F_{gravity} = m \cdot g = \rho \cdot V \cdot g \qquad (1)$$

Here V is the volume, ρ is the density of water, g is the acceleration due to gravity. Since the cube does not move, the gravity force is compensated by another force. What force compensates the gravity force? This compensating force is provided by the difference of water pressures applied to the upper and lower facets of the cube. The pressure applied to the lower facet is higher than the pressure applied to the upper facet; the resulting force pushes the cube up and compensates the gravity force. Zero net force means zero acceleration, so the water stays still. I choose “down to up” as positive direction.

$$-F_{gravity} + F_{lower} - F_{upper} = 0 \qquad (2)$$

F_{lower} and F_{upper} are the forces applied to the lower and upper facets of the cube. They can be expressed through the pressures as:

$$F_{lower} = P_{lower} \cdot S$$
$$F_{upper} = P_{upper} \cdot S \qquad (3)$$

Here S is the area of the cube facet. If we remember that $V=S\,l$, where l is the length of the cube edge we can rewright the equation (2) as:

$$\underbrace{-\rho \cdot g \cdot S \cdot l}_{\text{gravity force}} + \underbrace{P_{lower} \cdot S}_{\text{pressure force to the lower facet}} - \underbrace{P_{upper} \cdot S}_{\text{pressure force to the upper facet}} = 0 \qquad (4)$$

Dividing both left and right parts of the equation to S we obtain:

$$P_{lower} - P_{upper} = \rho \cdot g \cdot l \qquad (5)$$

So, as we can see, the difference in pressures applied to the lower and the upper facet depends only on the difference in the facet depths, l, and on the density of the liquid (water in our case). This formula is good for any liquid, for any glass shape. The pressure in liquid increases proportionally to the depth h:

$$P = \rho \cdot g \cdot h \qquad (6)$$

Problems:

1. Explain (qualitatively) why a balloon, filled with helium or hydrogen will soar if you let it go, while a balloon filled with argon or air will fall down the floor.
2. Calculate pressure near the bottom of a 40cm deep bucket with mercury. The density of mercury is 13.56 g/cm^3.