Homework 28.

Carnot cycle and theorem.

We discussed cyclic processes or cycles in the ideal gas. A cycle process underlies operation of any thermal engine. For example, the cycle used in the engine of a car using gas as the fuel is called Otto cycle; diesel cars use Diesel cycle, etc. But there is one special cycle which I would like to discuss. It is called Carnot cycle, named after French engineer and physicist Sadi Carnot.

Before we start discussing Carnot cycle, I would like to introduce another gas process. We know 3 basic gas processes: *isobaric* (the gas pressure does not change, or P=const), *isochoric* (the gas volume does not change, or V=const) and isothermal (the gas temperature does not change, or T=const). The new process is called *adiabatic*. In this process, all three parameters: pressure, volume and temperature are changing, but the gas is thermally isolated from the environment, i.e. the gas cannot receive or give away thermal energy. For adiabatic process $\Delta Q=0$. One example of adiabatic process is compressing the gas in a thermally insulated cylinder.

We know how does a isothermal process look in coordinates P-V. It is shown as a black curve in Figure 1. As log as we expand or compress the gas so that the pressure is proportional to reciprocal volume, the temperature of the gas does not change and the process is isothermal. But the gas does exchange the heat with the environment during the isothermal process. For example, we have to transmit heat energy to the gas to maintain its temperature constant during isothermal expansion. In adiabatic process the pressure is proportional not to 1/V, but $1/(V^{\gamma})$, $\gamma=C_P/C_V$. Here C_P and C_V are specific heat at constant pressure and constant volume.

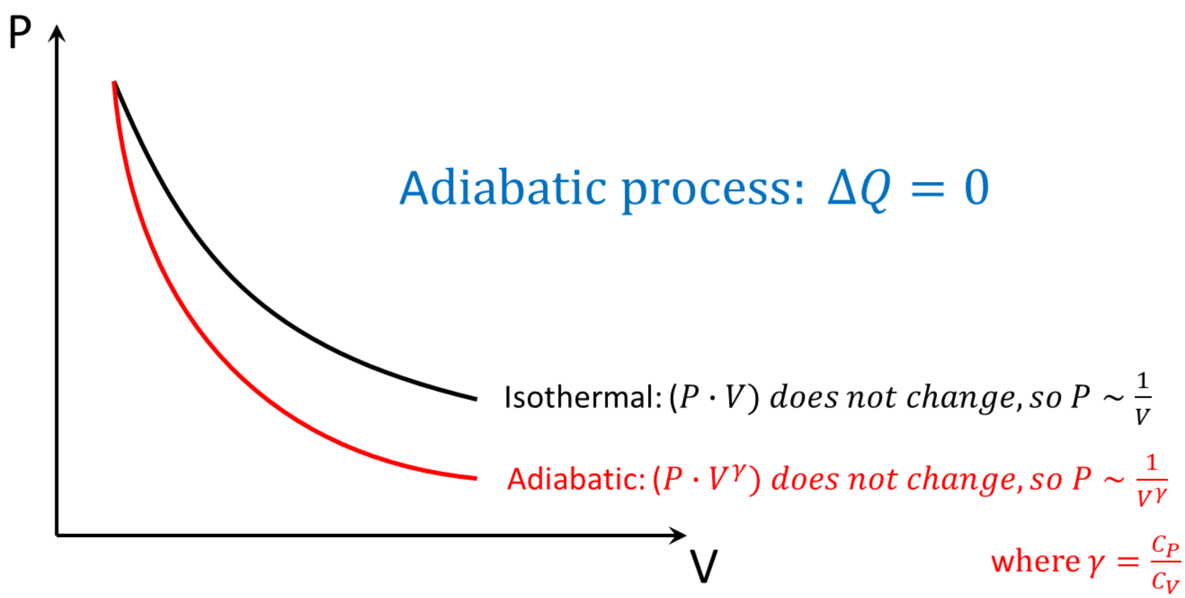
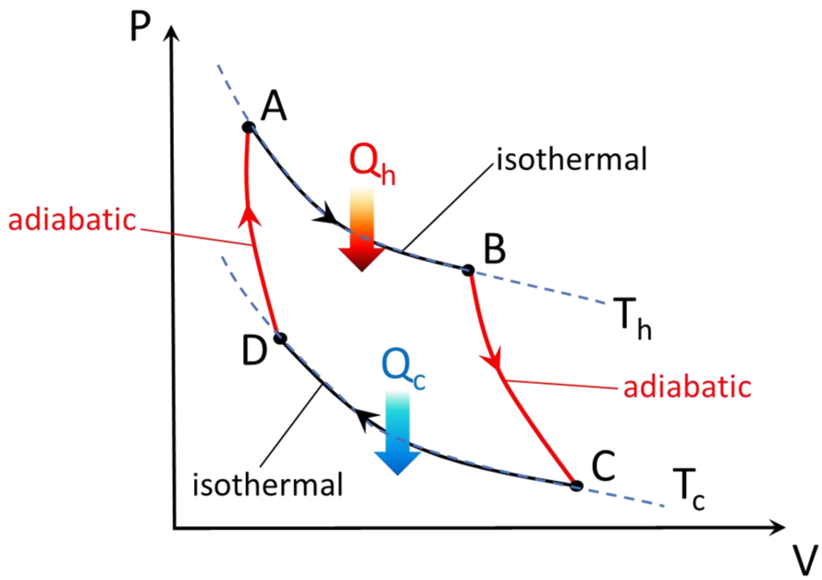


Figure 1. Isothermal and adiabatic processes.

It is not difficult to prove that but the proof requires a bit more advanced mathematics, so I omit it here. Adiabatic process looks a bit similar to isothermal in coordinates P-V, but adiabatic curve is more steep (red curve in Figure 1.

Now we are ready to discuss Carnot cycle. It is shown schematically in Figure 2.



Sadi Carnot (1796-1832)

Figure 2. Carnot cycle.

This cycle describes the operation of a theoretical engine which is called Carnot engine. It is named after French engineer Sadi Carnot.

The Carnot cycle includes four processes, as we can see in Figure 1.

1. Process AB is an isothermal expansion at temperature T_h (h means "heater"). During this process, the gas performs work (expands) and obtains heat Q_h from the heat reservoir, or , simply, the heater.
2. In process BC the gas expands adiabatically (it is thermally isolated and receives no heat). During this process the temperature falls from T_h to T_c (c means "cooler").
3. Process CD. The gas is compressed isothermally at temperature T_c. During this process the work is done on the gas, but the temperature of the gas remains constant since we simultaneously transfer heat Q_c from the gas to the cooler.
4. Process CA. The gas is compressed like in process CD but now it is thermally isolated. The work is done on the gas and the gas is heated up to temperature T_h.

It is very important that all four processes are performed very slow, so the gas reaches equilibrium at any point of the cycle.

Let us define the thermal efficiency of an engine as the ratio of work done by the engine during the cycle to the amount of heat which was "fed" to the engine during the cycle. The efficiency can not be higher than 1 due to energy conservation. The closer the efficiency to 1 the better the engine. Carnot demonstrated that the thermal efficiency of an engine based on the Carnot cycle is:

$$\eta_{carnot} = \frac{T_h - T_c}{T_h} = 1 - \frac{T_c}{T_h}$$

Moreover, Carno demonstrated that ***no real engine operating with the same heater and cooler temperatures can have efficiency higher than the Carnot engine.*** (This statement is called "Carnot theorem").

Problems:

1.The boiler of a steam engine operates at 600K. The steam pushes the piston and gets exhausted into the outside air at the temperature 300K. What is the maximum thermal efficiency of the engine?

2. The highest theoretical efficiency of the gasoline engine based on a Carnot cycle is 30%. The engine expels its gases into the atmosphere which has a temperature of 300K. What is the temperature of the cylinder immediately after the combustion?.