

## Homework 27

### Graphical representation of the work done by the ideal gas.

During the recent class we discussed one of the homework problems:

*280 g of gas is being heated at a constant pressure. The amount of heat supplied to the gas is 600J. Specific heat at a constant pressure  $C_P = 745 \text{ J/kgK}$ . Find the change of the gas's temperature.*

Solution of this problem is pretty simple. We just have to remember how to express the heat  $\Delta Q$  obtained by the gas through the change of the gas temperature  $\Delta T$ :

$$\Delta Q = m \cdot C_P \cdot \Delta T$$

Here  $m$  is the mass of the gas and  $C_P$  is the specific heat at the constant pressure. Plugging in the numbers we can calculate  $\Delta T$ :

$$\Delta T = \frac{\Delta Q}{m \cdot C_P} = \frac{600\text{J}}{0.28 \text{ kg} \cdot 745 \text{ J/kg} \cdot \text{K}} \approx 2.88\text{K}$$

Now, let us find what part of the heat obtained by the gas was spent to the work. To understand better what is going on let us represent the gas heating graphically:

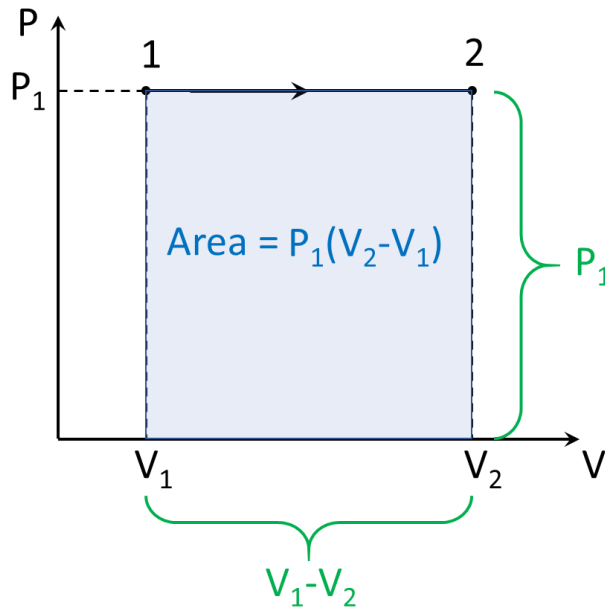


Figure 1.

At the plot, shown in Figure 1, point 1 represents the initial state of the gas, point 2 represents the final state. It is given that the gas pressure is maintained constant, so we have to increase the volume of the gas as we heat the gas. So the heating process is shown as a horizontal straight line connecting points 1 and 2.

We know that as long as the gas pressure is non-zero and the volume of the gas is changing that the work is done. As the pressure does not change, the work  $\Delta W$  can be calculated as pressure  $P_1$  multiplied by the change of the gas volume:  $\Delta W = P_1 (V_2 - V_1) = P_1 \Delta V$ . We do not know neither the pressure nor the change of the gas volume, but we can calculate the work using the equation of state of the ideal gas:

$$P \cdot V = n \cdot R \cdot T,$$

Where  $n$  is the number of moles of the gas and  $R$  is the Mendeleev-Clapeyron constant:  $R=8,31\text{J}/(\text{mole K})$ . Since the pressure does not change and is equal to  $P_1$ , we can write:

$$\Delta W = P_1 \cdot \Delta V = n \cdot R \cdot \Delta T$$

Let us assume that the gas is nitrogen ( $\text{N}_2$ ) with the molar mass of 28 g/mole. Then  $n=10$  and the work is:

$$\Delta W = n \cdot R \cdot \Delta T = 10 \cdot 8.31 \left[ \frac{\text{J}}{\text{mole} \cdot \text{K}} \right] \cdot 2.88\text{K} \approx 239.3\text{J}$$

So, 239.3J out of 600J is the work, done by the gas; the rest 360.7 J contributed to increase of the gas internal energy. The latter depends on the gas temperature.

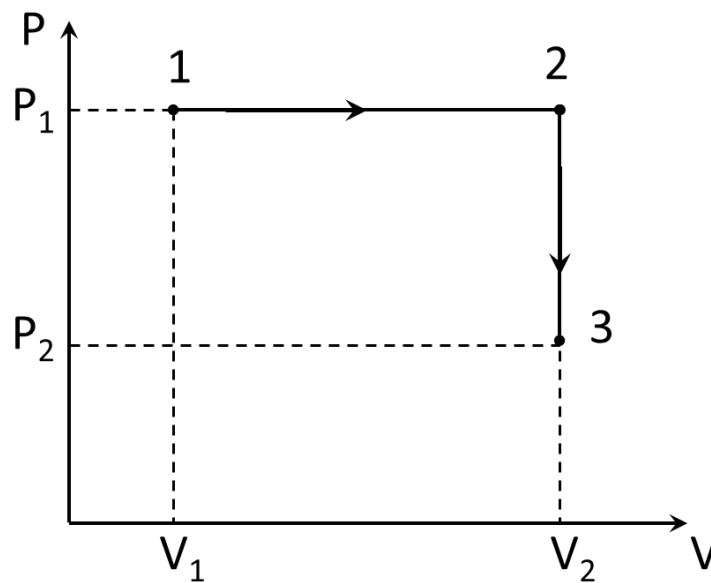


Figure 2.

Work done by the gas can be conveniently shown as the area under the line 1-2 (Figure 1). Now being in state “2” let us decrease the pressure to  $P_2$  while maintaining constant volume  $V_2$ . We can do that by reducing the temperature. This process is shown as line 2-3 in Figure 2.

Then we can compress the gas while maintaining the constant pressure  $P_2$  and reduce the volume to  $V_1$  again. Normally, when we compress the gas we can expect the pressure increase. But we can maintain constant pressure by cooling the gas while compressing it. This is the process 3-4 in Figure 3:

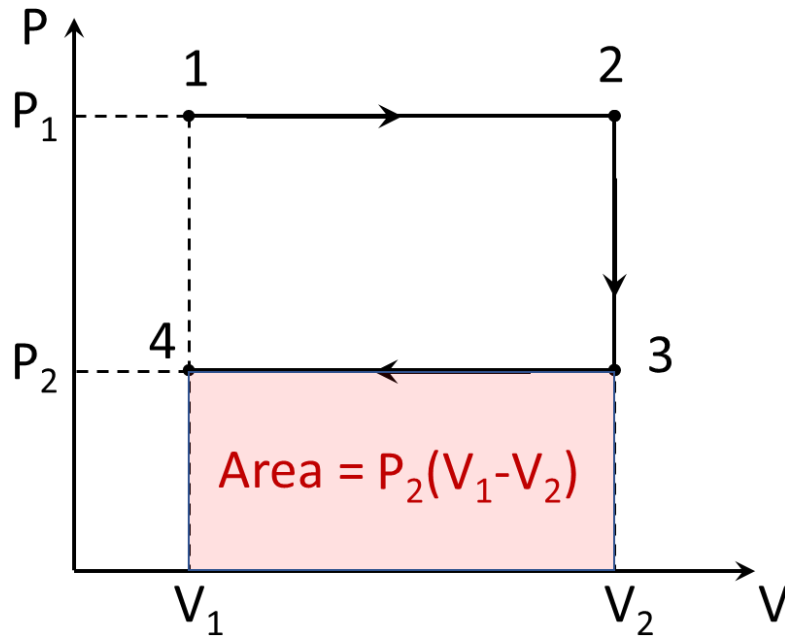


Figure 3.

During process 3-4 we do work *on the gas* by compressing it. Please notice that the change of the volume is negative: the destination volume  $V_1$  is lower than the initial volume  $V_2$ . So we can write the work  $\Delta W_{34}$  of the gas during process 3-4 as negative:

$$\Delta W_{34} = -P_2 \cdot (V_2 - V_1) = -P_2 \cdot \Delta V$$

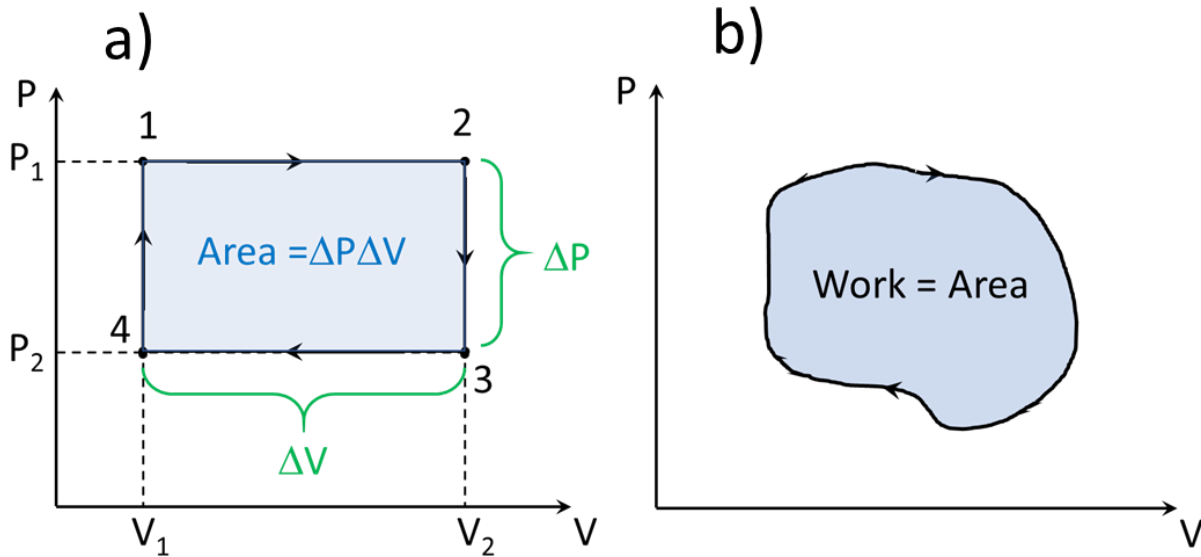
Absolute value of this work is equal to the pink-shaded area in Figure 3.

Then we will “return” to the initial point by heating the gas at constant volume  $V_1$  (process 4-1 in Figure 4). So we “moved” the gas state through the states 1-2-3-4-1 and returned to the initial state. Such process is called “cyclic process” or “cycle”.

Let us calculate the total net work  $W$ , done by the gas during this cycle:

$$W = \Delta W_{12} + \Delta W_{23} + \Delta W_{34} + \Delta W_{41}$$

Please note that  $\Delta W_{23} = \Delta W_{41} = 0$ , since the volume during these processes does not change and the work is not done. So, we have:



$$W = \Delta W_{12} + \Delta W_{34} = P_2 \Delta V - P_1 \Delta V = \Delta P \cdot \Delta V$$

But this work is numerically equal to the area of the rectangle 1-2-3-4 (Figure 4a):

Figure 4.

It turns out that for any closed process, work done by the gas is numerically equal to the area within the cycle process line, plotted in coordinates P-V (figure 4b)

Questions:

1. Which one is higher for a given gas: specific heat at a constant pressure  $C_p$  or specific heat at a constant volume  $C_v$ ? Try to prove your answer.
2. Will the total work for the process shown in Figure 4a change if we “pass” it counter-clockwise: 1-4-3-2-1?