

## MATH 9: FINAL PUZZLE

2021/05/23

### SUBMARINE!

- It is a fine day on the Integer River, and you are an unpleasant submarine hunter!
- The Integer River is an infinitely long, straight river, with regularly placed signposts that are marked with integers, as one might expect.
- There is a submarine peacefully traveling along the Integer River. It is an integer submarine! It began its journey at one of the signposted locations, and since then has been traveling at an integer speed per minute. For example, maybe it's traveling  $+2$  signposts per minute. Or maybe it's traveling  $-8$  signposts per minute.
- Every minute, on the minute, you may drop an underwater depth charge at a single signposted location. If the submarine is there at that moment, the submarine detonates, and you succeed at your hunting trip.
- Come up with a strategy to hunt the submarine. You do not know where the submarine started or what its speed is, and you cannot see it or sense it. But, you do have an unlimited supply of depth charges (and an unlimited lifespan). You also move infinitely fast, so you can go anywhere you want with your depth charges (it's setting up and detonating the charge that takes time). You begin at time  $t = 0$  and signpost 0; you may drop a bomb there and then if you wish, to start off the hunt.

## HINTS

1. It is possible to come up with a strategy that will find the submarine for certain, though you won't know exactly long it will take.
2. Can you solve the problem if you *do* know where the submarine starts and what its speed is?
3. Can you solve the problem if you know where the submarine starts but not what its speed is?
4. Let  $k$  be the submarine's starting location and  $s$  be its speed. For which values of  $k, s$  can the submarine be hunted, and where should you drop the depth charge at time  $t = 1$ ?
5. Can you solve the problem if you know the submarine's start point and speed, but you start at a time other than zero, for example time  $t = m$  for some integer  $m > 0$ ?
6. Let  $k \in \mathbb{Z}$  be the submarine's starting location and  $s \in \mathbb{Z}$  be its speed. Make a table of all possible values of  $k$  and  $s$ . Is it possible to fill each cell of this table with a different natural number?
7. Not a hint but, have a wonderful summer :)