## MATH 9: REVIEW 2

2021/04/25

## 1. Complex Numbers

The basics of complex numbers:

- The set of complex numbers is written with the symbol $\mathbb{C}$. As a reminder, $\mathbb{R}$ is the real numbers.
- Complex numbers are found by taking roots of polynomials with real coefficients.
- Or, complex numbers are found as combinations of the real and imaginary unit numbers 1 and $i$, using real coefficients: $a \cdot 1+b \cdot i$ for $a, b \in \mathbb{R}$. The real numbers $a, b$ are called the real part and the imaginary part. This is called the Cartesian form.
- The conjugate of a complex number is what you get if you replace $i$ with $-i$. The complex conjugate of $a+b i$ is $a-b i$. Conjugates are written with a bar: if $z=a+b i$, then $\bar{z}=a-b i$.
- If you multiply a complex number by its conjugate, you get a real number.
- The sum of the squares of the real and imaginary part is equal to the number times its conjugate. This parallel of the Pythagorean Theorem hints that the complex numbers may follow some geometry rules if we draw them on a plane. In fact, it turns out they make a nice coordinate plane: this is usually drawn with the real part on the $x$-axis and the imaginary part on the $y$-axis.
- More to come next week!


## 2. Problems

1. Given numbers $m=a+b i$ and $n=c+d i$, find the real and imaginary part of:
(a) $m+n$
(b) $m n$
2. (a) Compute $(1+i)^{2}$. Write it in Cartesian form.
(b) Use your result from the previous part to determine $\sqrt{i}$. It is a complex number, and can be written in Cartesian form, with a real and imaginary part. (As usual with square roots, the result is unique up to multiplication by $\pm 1$.
3. An infinite subset of $\mathbb{N}$ has infinitely many elements; a coinfinite set of $\mathbb{N}$ is a set $A$ such that $\mathbb{N} \backslash A$ is infinite. For this problem, assume that all sets are subsets of $\mathbb{N}$.
(a) Find an infinite set that is not coinfinite.
(b) Find a set that is both infinite and coinfinite.
(c) Find two sets that are both infinite and coinfinite, whose union and intersection are also both infinite and coinfinite.
4. Find a collection $C_{0}, C_{1}, C_{2}, \ldots$ of infinite subsets of $\mathbb{Z}$ such that the intersection of any two of the sets is infinite, but the intersection of all of them is empty.
5. Prove that the altitudes of a triangle are the angle bisectors of another triangle, whose vertices are the feet of the altitudes of the first triangle.
6. Let $\triangle A B C$ be a triangle, whose altitudes intersect at point $O$. Find the intersections of the altitudes of triangle $\triangle B C O$.
7. (a) Find the minimum possible value of $x(1-x)$, for $x \in \mathbb{R}$. What is the value of $x$ that achieves this minimum?
(b) Prove that this value is the minimum by proving that, $\forall y \in \mathbb{R}, y(1-y) \geq x(1-x)$, for the value of $x$ you found in the previous part.
