

MATH 8: HANDOUT 27

REVIEW TEST

1. Compute

$$(1 + \sqrt{5})^7 + (1 - \sqrt{5})^7$$

2. Prove distributive laws using truth tables:

$$A \vee (B \wedge C) \leftrightarrow (A \vee B) \wedge (A \vee C)$$

$$A \wedge (B \vee C) \leftrightarrow (A \wedge B) \vee (A \wedge C)$$

3. Write the following statements using logic operations and quantifiers:

- (a) All linguists speak Swahili
- (b) Some linguists don't speak Swahili
- (c) No one but a linguist speaks Swahili
- (d) No one would wake up at 5am unless they are a linguist or they speak Swahili

Please use the following notation:

P – set of all people

$L(x)$ — x is a linguist

$S(x)$ — x speaks Swahili

$W(x)$ — x wakes up at 5am

4. (a) Compute $\gcd(32, 12)$ **using Euclid's algorithm**
(b) Write $\gcd(32, 12)$ in the form $32k + 12l$. (You can use guess and check, or proceed in the same way as in the previous problem)
(c) Does the equation $32x + 12y = 10$ have integer solutions? Can you find at least one solution?
(d) Does the equation $32x + 12y = 11$ have integer solutions? Can you find at least one solution?
(e) Can you give complete answer, for which integer values of c the equation $32x + 12y = c$ has integer solutions?
5. (a) Find an inverse of $13 \pmod{23}$.
(b) Solve an equation $13x \equiv 5 \pmod{23}$.

6. Find all solutions of the system:

$$x \equiv 4 \pmod{5}$$

$$x \equiv 3 \pmod{7}$$

$$x \equiv 3 \pmod{11}$$