MATH 7: HANDOUT 24
TRIGONOMETRY 6: GRAPH OF TANGENT. MORE EQUATIONS.

TANGENT IN THE TRIGONOMETRIC CIRCLE

Just like with the sine and the cosine, the tangent can be defined through the trigonometric circle. Given a point (with coordinates $(\cos\alpha, \sin\alpha)$) corresponding to the angle α, one draws a line which passes through the origin and through this point, the tangent of α is the position along a line tangent to the circle through the point $(1, 0)$ of the intersection of these two lines (see figure 1). For angles $0 < \alpha < \pi/2$, this agrees with the previous. For other angles, we will use this procedure to define the tangent.

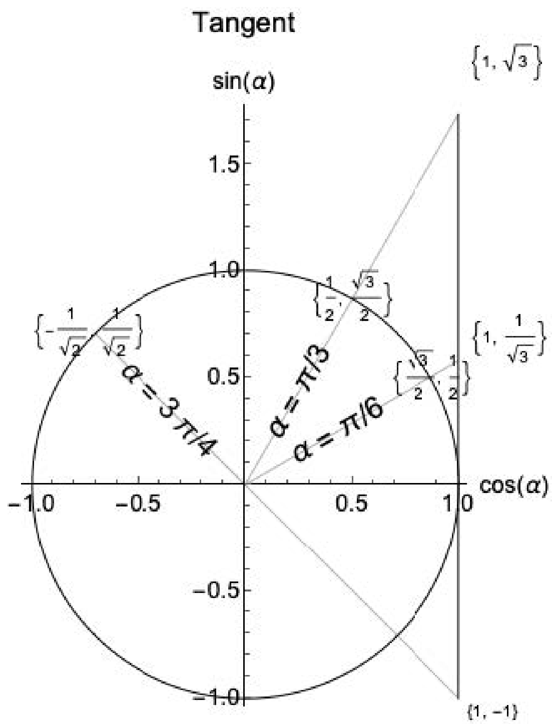

FIGURE 1. Meaning of $\tan\alpha$ in the trigonometric circle.

Compare this definition with the values we know from the table:

Trigonometric Functions							
Function	Notation	Definition	0	$\frac{\pi}{6}/30°$	$\frac{\pi}{4}/45°$	$\frac{\pi}{3}/60°$	$\frac{\pi}{2}/90°$
sine	$\sin(\alpha)$	$\frac{\text{opposite side}}{\text{hypotenuse}}$	0	$\frac{1}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{3}}{2}$	1
cosine	$\cos(\alpha)$	$\frac{\text{adjacent side}}{\text{hypotenuse}}$	1	$\frac{\sqrt{3}}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{1}{2}$	0
tangent	$\tan(\alpha)$	$\frac{\text{opposite side}}{\text{adjacent side}}$	0	$\frac{1}{\sqrt{3}}$	1	$\sqrt{3}$	∞

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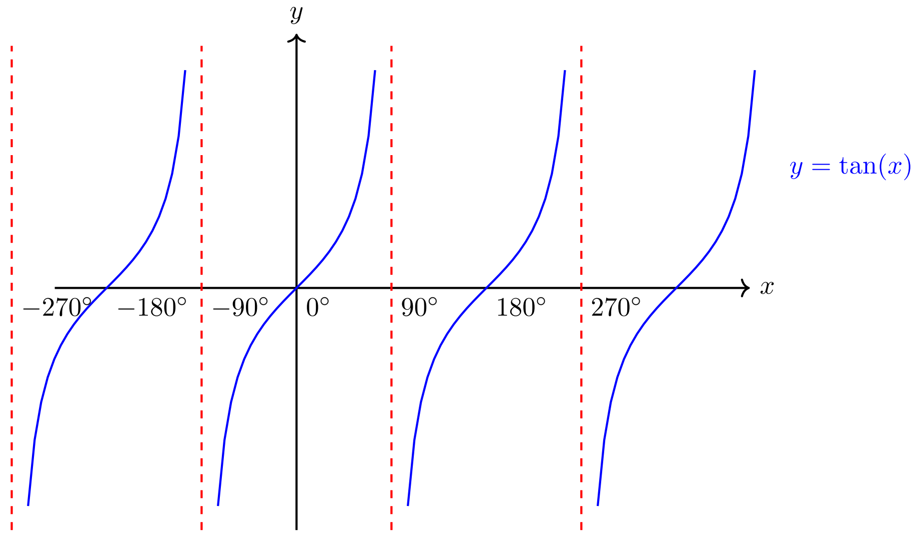
GRAPH OF TANGENT

By looking at the values of tangent as we go around the trigonometric circle, we find out a few facts like:

- $\tan 0° = \tan 180° = 0$
- $\tan x$ increases from 0 to $90°$.

- As x is approaching $90°$, $\tan x$ grows all the way to "infinity."
- As x goes from $0°$ to $-90°$, $\tan x$ decreases all the way to "negative infinity."
- $\tan x + 180° = \tan x$.

We can see all of these facts clearly in the graph of the function $\tan x$:



Solving the equation $\tan x = \tan c$. In this case (figure 2) we find that, for a given c, the solution to the equation $\tan x = \tan c$ is

$$x = c + \pi \times n$$

or

$$x = c + 180° \times n,$$

where n can be any integer number.

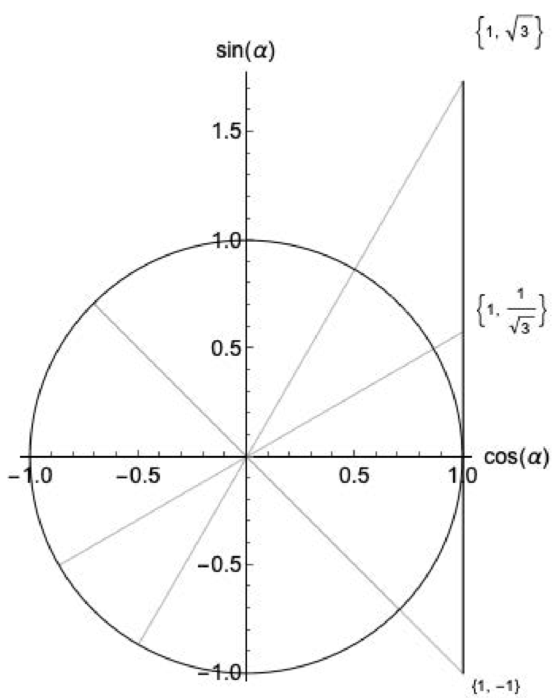


FIGURE 2. A few examples of the relation $\tan x = \tan(x + \pi)$.

Law of cosines

The law of cosines allows you to find the third side of a triangle if you know two sides and an angle between them. That is, if a and b are two sides, and $\angle C$ is an angle between them, then the third side c can be found using the following formula:

$$c = \sqrt{a^2 + b^2 - 2ab \cdot \cos \angle C}$$

Homework

1. Solve the following equations
  (a) $\sqrt{3} \tan x = 1$
  (b) $\tan x = -1$
  (c) $\tan x = -\sqrt{3}$
  (d) $(\sin x)^2 = (\cos x)^2$ [Hint: $\tan x = \sin x / \cos x$.]

2. Determine the internal angles of a triangle ABC knowing that the angles form an arithmetic sequence and that the sine of the sum of the two smallest angles is $\sqrt{3}/2$.

3. What are the values of x for which:
  (a) $\sin x > -\frac{\sqrt{2}}{2}$
  (b) $-\frac{1}{2} \leq \sin x < \frac{\sqrt{2}}{2}$
  (c) $2(\sin x)^2 < \sin x$
  (d) $\cos 2x \leq \frac{\sqrt{3}}{2}$
  (e) $(\tan x^2)^2 \leq 1$

  It might help to examine the graphs of trigonometric functions to answer this question.