MATH 6: TRANSFORMATIONS

1. Symmetries and Transformations

A transformation is an operation which sends every point (x, y) of the plane to a new point (x', y').

A transformation is a rigid motion or an isometry if it preserves distances: for any points P, Q, we have $PQ = P'Q'$.

Theorem 1.

1. *An isometry preserves angles.*
2. *An isometry preserves parallelism.*
3. *An isometry preserves areas.*
4. *Any isometry sends lines to lines: if l is a line and T an isometry, then $T(l)$ is again a line.*
5. *Composition of isometries is again an isometry.*

Here are some examples of transformations:

Reflection: For any line l, the reflection r_l is defined by the condition that the reflection P' of P lies on the perpendicular from P to l, on the other side of l than P, at the same distance from l: in other words, l is the perpendicular bisector of $P'P$. The notation is $r_l(P) = P'$

Rotation: For any point O and real number φ, we define rotation $R_{O,\varphi}$ to be the counterclockwise rotation around point O by the angle φ (if φ is negative, clockwise rotation by angle $|\varphi|$) . The notation is $R_{O,\varphi}(P) = P'$

Translation: A translation is a transformation that slides or moves every point of a figure by the same distance in a given direction. The notation we use is $T_{a,b}(P) = P'$, where a and b represent the units on the x-axis and respectiv y-axis by which the point moves. In other words if $P(x, y)$ then $P(x', y')$ has coordinates $x' = x + a =$ and $y' = y + b$. For example $T_{2,-3}(1, 3) = (2 + 1, -3 + 3) = (3, 0)$

Theorem 2. *Reflections, rotations, and translations are isometries.*

2. Symmetry

A figure has line symmetry if it can be folded in half and every point in one half maps onto a ppoint in the second half.

A figure has rotational symmetry if the figure can be rotated by a given angle and every point on the rotated figure maps to a point on the original figure.

Homework

1. What is the image of point P(5,-1) after a rotation of 180°
2. Given $P(2, 3)$, what are the coordinates of $T_{2,6}(P)$?
3. $S' = r_{y-axis}(S)$. What are the coordinates of S' if S has coordinates $(-1, 4)$?
4. What are the new coordinates of point (x,y) after a rotation of 90°? (Or using the new notation we learned, what is $R_{90}(x, y)$?) How about $R_{180}(x, y)$? $R_{270}(x, y)$?
5. The image of point L after translation $(x, y) \to (x + 3, y - 2)$ is $L'(5, 1)$. What are the coordinates of L?