

MATH 6: HANDOUT 27: INVARIANTS

An invariant is something that does not change...

In this class of problems you are given a set of objects (like some numbers) and then some operations that can be performed on these objects. The question is if a new object can be obtained from the given set of objects. For each problem, think about an invariant (a rule, an expression) that doesn't change when the operations are performed. Does the new object satisfy the invariant rule? If no, then you proved that that the new object can not be obtained from the given set. Problems 1 to 3 are also invariant problems but you are asked to predict the final result.

1. Numbers 1 through 20 are written on the blackboard. Every minute two of the numbers are erased and replaced by their sum. Can you predict which number will be written on the board at the end?
2. Students have written on the blackboard 2011 "+" signs and 2011 "-" signs. Every minute a pair of signs is erased and replaced by a single "+" if they were equal or a single "-" if they were different. Can you predict which sign will be written on the board at the end?
3. Numbers 1 through 20 are written on the blackboard. Every minute a pair of numbers a, b are erased and replaced by $a + b - 1$. Can you predict which number will be written on the board at the end?
4. In the alphabet used by the tribe OUO there are only two letters, O and U. Two words in their language are synonyms if one word can be obtained from the other by crossing out or adding anywhere in the word the combinations "OU" and "UOO". Are the words OOU and UOO synonyms?
5. There are 16 glasses on the table, one of them upside down. You are allowed to turn over any 4 glasses at a time. Can you get all glasses standing correctly by repeating this operation?
6. In the country of RGB, there are 13 red, 15 green and 17 blue chameleons. Whenever two chameleons of different colors meet, both of them change their color to the 3rd one (e.g., if red and green meet, they both turn blue). Do you think it can happen that after some time, all chameleons become the same color? [Hint: give each color a numeric value, say 0, 1, 2]