

MATH 6: HANDOUT 26

SQUARE ROOTS

The square root of a number a is a number whose square is equal to a . For example the square root of 25 is 5, because $5^2 = 25$

The square root of number a is commonly denoted by \sqrt{a} .
Note that $\sqrt{ab} = \sqrt{a}\sqrt{b}$, but $\sqrt{a+b}$ is not equal to $\sqrt{a} + \sqrt{b}$.
Another way to think about radicals is using exponents laws.

$$a^{\frac{1}{2}} = \sqrt[2]{a} = \sqrt{a}$$

$$a^{\frac{1}{3}} = \sqrt[3]{a}, \text{ this is called the cubic root of } a$$

$$a^{\frac{1}{n}} = \sqrt[n]{a}, \text{ in general this is called - the } n\text{-th root of } a \text{ (or } a \text{ radical } n)$$

1. REVIEW ALGEBRA

$$a^0 = 1$$

$$a^m a^n = a^{m+n}$$

$$\frac{a^m}{a^n} = a^{m-n}$$

$$\frac{a^n}{(ab)^n} = a^n b^n$$

$$\left(\frac{a}{b}\right)^n = \frac{a^n}{b^n}$$

$$a^{-n} = \frac{1}{a^n}$$

$$(a^m)^n = a^{mn}$$

$$\sqrt{ab} = \sqrt{a} \cdot \sqrt{b}$$

$$(a+b)^2 = a^2 + 2ab + b^2$$

$$(a-b)^2 = a^2 - 2ab + b^2$$

$$a^2 - b^2 = (a-b)(a+b)$$

Replacing a by \sqrt{a} and b by \sqrt{b} above, we get $(\sqrt{a} - \sqrt{b})(\sqrt{a} + \sqrt{b}) = a - b$

2. SIMPLIFYING SQUARE ROOTS

If you have a square root in the denominator, then you can use the following trick to simplify the expression:

$$\frac{1}{\sqrt{2}} = \frac{1}{\sqrt{2}} \times \frac{\sqrt{2}}{\sqrt{2}} = \frac{\sqrt{2}}{2}$$

We also discussed solving equations where the left hand side is factored as a product of linear factors such as $(x-1)(x+3) = 0$

3. HOMEWORK PROBLEMS

1. Compute without calculator 199×201 .

2. Find the following square roots.

(a) $\sqrt{16} =$

(b) $\sqrt{10,000} =$

- (c) $\sqrt{10^8} =$
- (d) $\sqrt{50} =$
- (e) $\sqrt{4a^3} =$
- (f) $\sqrt{81a^4b^7} =$

3. Simplify

- (a) $\sqrt{7}\sqrt{7}\sqrt{7}\sqrt{7}\sqrt{7} =$
- (b) $\sqrt[3]{3}\sqrt[3]{3}\sqrt[3]{3}\sqrt[3]{3}\sqrt[3]{3}\sqrt[3]{3} =$
- (c) $(\sqrt{17} - \sqrt{11})(\sqrt{17} + \sqrt{11}) =$
- (d) $(\sqrt{7} - \sqrt{2})(\sqrt{7} - \sqrt{2}) =$

4. Simplify

- (a) $7\sqrt{5} + 3\sqrt{5}$
- (b) $-5\sqrt{8} + 3\sqrt{8} - 2\sqrt{8} + 4\sqrt{8}$
- (c) $\frac{\sqrt{15}}{\sqrt{5}}; \sqrt{\frac{3}{10}} \cdot \sqrt{\frac{22}{21}}; \sqrt{\frac{7^2 \cdot 11}{5^4 \cdot 5^2}}$
- (d) $\sqrt{7x^2}; \sqrt{\frac{4x^4}{9y^2}}$

5. Simplify the following equations, writing them in the form $\frac{f}{g}$, where f,g are expressions in x .

- (a) $\frac{1}{x+1} - \frac{1}{x-1}$
- (b) $\left(1 + \frac{1}{x}\right) \div (x+1)$
- (c) $\left(1 + \frac{1}{x}\right) \div \left(1 - \frac{1}{x}\right)$

6. Factor (i.e. write as a product) the following expressions:

- (a) $a^2 + 4ab + 4b^2$
- (b) $a^2 - 2a + 1$
- (c) $a^4 - b^4$ [Hint: $a^4 = (a^2)^2$]
- (d) $x^2 - 7$ [Hint: $7 = (\sqrt{7})^2$]

7. Write each of the following expressions in the form $a + b\sqrt{3}$, with rational a, b :

- (a) $(1 + \sqrt{3})^2$

(b) $\frac{1}{\sqrt{3}}$

(c) $\frac{1 + \sqrt{3}}{\sqrt{3}}$

(d) $\frac{1 + 2\sqrt{3}}{\sqrt{3}}$

8. Solve the equation $(x - 1)^2 = 6$

*9. Is $\sqrt{2}$ a rational number? [Hint: Write $\sqrt{2} = \frac{m}{n}$, where m, n are irreducible. Then square it. Can you show that m and n are not irreducible?]