Math 5B: Classwork 26
Homework #26 is due May 2-nd

Beginning Probability Theory

We will be talking about "tests" or "events" such as tossing a coin, rolling a die, drawing a card, etc. Each event can result in one of several possible outcomes: for example rolling a die can give numbers 1 through 6. If there are *n* possible outcomes, and they are all equally likely, then the probability of obtaining any given one is exactly 1/*n*. For example, the probability of rolling a 3 on a die is 1/6.

In most cases, we will be interested in the probability of something that can be obtained in more than one way. For example, if we ask what is the probability of rolling an even number on a die, then there are 3 ways to obtain it: by rolling 2, 4, or 6. Each of these outcomes has the probability 1/6, so the probability of rolling one of them is 1/6 + 1/6 + 1/6 = 3/6 = 1/2.

In general, the probability of obtaining one outcome from a certain collection A of possible outcomes, is given by

$$P(A) = \frac{Number\ of\ outcomes\ giving\ A}{total\ number\ of\ possible\ outcomes}$$

Addition Rule

Suppose one draws a card from a regular deck of 52 cards, what is the probability of drawing either a queen or a king? Since there are 4 queens and 4 kings, there are a total of 8 cards for this outcome, we can write:

$$P(QorK) = \frac{4+4}{52} = \frac{8}{52} = \frac{2}{13}$$

We can also write it as follows:

$$P(QorK) = \frac{4+4}{52} = \frac{4}{52} + \frac{4}{52} = P(Q) + P(K)$$

In general, we have the following rule:

$$P(A\ or\ B) = P(A) + P(B)$$

Note: this is true if A and B cannot happen together. For example, there are 26 red cards in the deck, so the probability of drawing a red card is $\frac{26}{52} = \frac{1}{2}$. However, if we want a red card or a queen, then using addition formula would give $\frac{26}{52} + \frac{4}{52} = \frac{30}{52}$, which is incorrect. In this way, we have counted the red queens twice. Correct answer is $\frac{28}{52}$: 26 red cards plus two black queens (no need to count the red queens – they have already been counted).

Complement Rule

$$P(notA) = 1 - P(A)$$

For example, the probability of drawing a queen from a deck of cards is $\frac{1}{13}$ thus, the probability of drawing something ***other than*** a queen is $1 - \frac{1}{13} = \frac{12}{13}$

Homework

1. Write each of the following expressions in the form $2^n 5^k$

   (a) $\frac{2^2 5^8}{2^5 5^3}$ (b) $(2^3)^2 10^2 5^{-3}$ (c) $\frac{2^8 5^{-14}}{10^{-3}}$

2. In the game of roulette, there are 37 slots, numbered 0 through 36. Half of the numbers 1–36, half are red, the other half are black (zero has no color). What is the probability of hitting
   (a) A number between 1–12
   (b) An even number other than zero
   (c) A red number or zero
   (d) If you bet $15 on odd numbers (i.e., you win if you roll an odd number), what is the probability of losing?

3. The standard card deck has 4 suits (hearts, diamonds, spades, and clubs); each suit has 13 different card values: 2 through 10, jack, queen, king, and ace. If you randomly draw one card, what is the probability of drawing
   (a) The queen of spades
   (b) A face card (i.e., jack, queen, or king)
   (c) A black king
   (d) Anything but the queen of hearts

4. If I had drawn an ace from a deck of cards. What is the probability to randomly draw a second ace from the remaining cards?

5. What is the probability that a randomly chosen person was born
(a) in January?
(b) on Feb 5?
(c) on Sunday?
(d) On Sunday in January? [Hint: among all the people born in January, what fraction were born on Sunday?]

When doing this problem, you can ignore leap years and assume that birthdays are randomly distributed among all days of the year, so each day is equally likely; in real life it is not quite true.

6. Suppose we have a box of 500 candies of different colors and sizes. We know that there are 100 large ones and 400 small ones; we also know that there are 70 red ones, 11 of which are large. From this information, can you compute the probability that a randomly chosen candy will be either red or large (or both)?

7. When tossing a coin, it can land either with heads up or tails up (we will write H for heads and T for tails).
   (a) If we toss a coin 3 times, what is the probability that all 3 will be heads?
   (b) If we toss a coin 3 times, what is the probability of obtaining this sequence of results: HTH? Is it more likely or less likely than getting all 3 heads?

Hint: write out all possible combinations $\{HHH, HHT, \ldots\}$.