Homework for April 7, 2019.

Algebra/Trigonometry.

Please, complete the previous homework assignments from this year. Review the classwork handout on trigonometric functions. Additional reading on trigonometric functions is Read Gelfand & Saul, Trigonometry, Chapter 2 (pp. 42-54) and Chapters 6-8 (pp. 123-163),
http://en.wikipedia.org/wiki/Trigonometric_functions
http://en.wikipedia.org/wiki/Sine. Solve the following trigonometry problems repeated from the last assignment.

Problems.

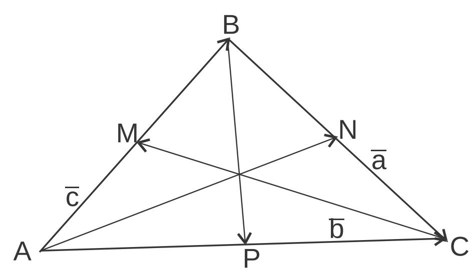
1. Solve the following equations and inequalities:
    a. $\sin x + \sin 2x + \sin 3x = \cos x + \cos 2x + \cos 3x$
    b. $\cos 3x - \sin x = \sqrt{3}(\cos x - \sin 3x)$
    c. $\sin^2 x - 2\sin x \cos x = 3\cos^2 x$
    d. $\sin 6x + 2 = 2\cos 4x$
    e. $\cot x - \tan x = \sin x + \cos x$
    f. $\sin x \geq \pi/2$
    g. $\sin x \leq \cos x$

Geometry.

Please, complete the previous homework assignments from this year. Review the classwork handout on vectors. Solve the following problems.

Problems.

1. In a triangle ABC, vectors $\overrightarrow{AB}$, $\overrightarrow{AC}$ and $\overrightarrow{BC}$ (**c**, **b** and **a**) are the sides. $\overrightarrow{AN}$, $\overrightarrow{CM}$ and $\overrightarrow{BP}$ are the medians.



    a. Express vectors $\overrightarrow{AN}$, $\overrightarrow{CM}$ and $\overrightarrow{BP}$ through vectors **c**, **b** and **a.**
    b. Find the sum of vectors $\overrightarrow{AN}$, $\overrightarrow{CM}$ and $\overrightarrow{BP}$**.**

2. Solve the same problem for bisectors $\overrightarrow{AN}$, $\overrightarrow{CM}$ and $\overrightarrow{BP}$ in a triangle ABC.

3. Coxeter, Greitzer, problem #9 to Sec. 2.1 (p. 31): How far away is the horizon as seen from the top of a mountain 1 mile high? (Assume the Earth to be a sphere of diameter 7920 miles.)

4. In a rectangle $ABCD$, A_1, B_1, C_1 and D_1 are the mid-points of sides AB, CD, BC and DA, respectively. M is the crossing point of the segments A_1B_1, and C_1D_1, connecting two pairs of midpoints.

   a. Express vector $\overrightarrow{A_1M}$ through $\overrightarrow{AB}$, $\overrightarrow{BC}$ and $\overrightarrow{CD}$.

   b. Prove that M is the mid-point of segments, A_1B_1 and C_1D_1, i.e. $|A_1M| = |MB_1|$ and $|C_1M| = |MD_1|$.

5. In a parallelogram $ABCD$, find $\overrightarrow{AB} + \overrightarrow{BD} - 2\overrightarrow{AD}$.

6. M is a crossing point of the medians in a triangle ABC. Prove that $\overrightarrow{AM} = \frac{1}{3}\left(\overrightarrow{AB} + \overrightarrow{AC}\right)$.

7. For three points, $A(-1,3)$, $B(2,-5)$ and $C(3,4)$, find the (coordinates of) following vectors,

   a. $\overrightarrow{AB} - \overrightarrow{BC}$

   b. $\overrightarrow{AB} + \overrightarrow{CB} + \overrightarrow{AC}$

   c. $\overrightarrow{AB} + \frac{1}{2}\overrightarrow{BC} + \frac{1}{3}\overrightarrow{CA}$

8. For two triangles, ABC and $A_1B_1C_1$, $\overrightarrow{AA_1} + \overrightarrow{BB_1} + \overrightarrow{CC_1} = 0$. Prove that medians of these two triangles cross at the same point M.