Homework for March 17, 2019.

Algebra.

Review the classwork handout. Complete the unsolved problems from the previous homework and classwork exercises. Solve the following problems.

1. Let x_1, x_2 and x_3 be distinct real numbers. Prove that there exists a unique polynomial, $P(x)$, of degree 2 such that $P(x_1) = 1$, $P(x_2) = P(x_3) = 0$. [Hint: if $P(x_1) = 0$, then $P(x)$ is divisible by $(x - x_1)$.] Find this polynomial if $x_1 = 2$, $x_2 = -1$, $x_3 = 5$.
2. As before, let x_1, x_2 and x_3 be distinct real numbers, and let y_1, y_2 and y_3 be any collection of numbers. Prove that there is a unique quadratic polynomial $f(x)$ such that $f(x_1) = y_1$, $f(x_2) = y_2$, $f(x_3) = y_3$. Find this polynomial if $x_1 = 2$, $x_2 = -1$, $x_3 = 5$, $y_1 = 3$, $y_2 = 6$, $y_3 = 18$. [Hint: look for in the form $f(x) = y_1 f(x_1) + \cdots$.]
3. Prove the following general result: given numbers $x_1, \dots, x_n, y_1, \dots, y_n$, such that x_i are distinct, there exists a unique polynomial $f(x)$ of degree $n - 1$ such that $f(x_i) = y_i$, $i = 1, \dots, n$. (For $n = 2$, this is a statement that there is a unique line through two given points.)
4. Prove that if $P(x)$ is a polynomial with integer coefficients, then for any integer a, b, the difference $P(a) - P(b)$ is divisible by $a - b$.
5. Le t x_1 and x_2 be the roots of the polynomial, $x^2 + 7x - 3$. Find
    a. ${x_1}^2 + {x_2}^2$
    b. $\frac{1}{x_1} + \frac{1}{x_2}$
    c. $(x_1 - x_2)^2$
    d. ${x_1}^3 + {x_2}^3$

Geometry/Trigonometry.

Read the classwork handout. Complete the unsolved problems from the previous homework. Additional reading on trigonometric functions is Gelfand & Saul, Trigonometry, Chapter 2 (pp. 42-54) and Chapters 6-8 (pp. 123-163), http://en.wikipedia.org/wiki/Trigonometric_functions http://en.wikipedia.org/wiki/Sine. Solve the following problems.

1. Simplify the following expressions:
   a. $\frac{\sin(\pi+\alpha)\cos(\pi-\alpha)}{\sin(\alpha-\pi)\cos(\alpha+\pi)}$
   b. $\frac{\cot^2\left(\alpha+\frac{\pi}{2}\right)\cos^2\left(\alpha-\frac{\pi}{2}\right)}{\cot^2\left(\alpha-\frac{\pi}{2}\right)-\cos^2\left(\alpha+\frac{\pi}{2}\right)}$
   c. $\frac{\cot\left(\frac{3\pi}{2}-\alpha\right)}{1-\tan^2(\alpha-\pi)}\cdot\frac{\cot^2(2\pi-\alpha)-1}{\cot(\alpha+\pi)}$
   d. $\frac{\cos^2\left(\alpha-\frac{3\pi}{2}\right)}{\sin^{-2}\left(\alpha+\frac{\pi}{2}\right)-1}\cdot\frac{\sin^2\left(\alpha+\frac{3\pi}{2}\right)}{\cos^{-2}\left(\alpha-\frac{\pi}{2}\right)-1}$
   e. $\frac{\left(1+\tan^2\left(\alpha-\frac{\pi}{2}\right)\right)\left(\sin^{-2}\left(\alpha-\frac{3\pi}{2}\right)-1\right)}{\left(1+\cot^2\left(\alpha+\frac{3\pi}{2}\right)\right)\cos^{-2}\left(\alpha+\frac{\pi}{2}\right)}$
   f. $\frac{\sin^2\left(\alpha+\frac{\pi}{2}\right)-\cos^2\left(\alpha-\frac{\pi}{2}\right)}{\tan^2\left(\alpha+\frac{\pi}{2}\right)-\cot^2\left(\alpha-\frac{\pi}{2}\right)}$
2. Solve the following equations (find all solutions):
   a. $\sin x = \frac{1}{2}$
   b. $\tan x = 1$
   c. $\cos x = \frac{\sqrt{3}}{2}$
   d. $\cos^2 x = \frac{1}{2}$