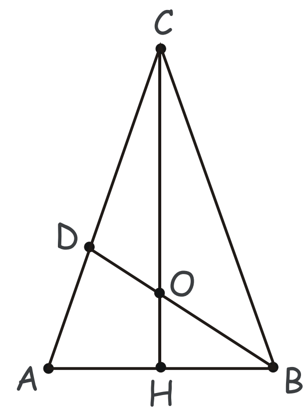
Homework for November 4, 2018.

Geometry.

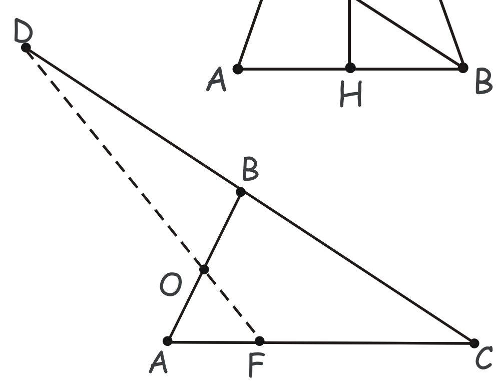
Review the classwork handout. Solve the unsolved problems from previous homeworks. Try solving the following problems from the previous homework using the method of point masses and the Law of Lever.

Problems.

1. Prove that if a polygon has several axes of symmetry, they are all concurrent (cross at the same point).
2. Prove that medians of a triangle divide one another in the ratio 2:1, in other words, the medians of a triangle "trisect" one another (Coxeter, Gretzer, p.8).
3. In isosceles triangle ABC point D divides the side AC into segments such that $|AD|:|CD|=1:2$. If CH is the altitude of the triangle and point O is the intersection of CH and BD, find the ratio $|OH|$ to $|CH|$.



4. Point D belongs to the continuation of side CB of the triangle ABC such that $|BD| = |BC|$. Point F belongs to side AC, and $|FC| = 3|AF|$. Segment DF intercepts side AB at point O. Find the ratio $|AO|:|OB|$.



Algebra.

Review the classwork handout. Solve the remaining problems from the previous homework assignments and classwork exercises. Try solving the following problems.

1. Verify that a set of eight numbers, {1,2,3,5,6,10,15,30}, where addition is identified with obtaining the least common multiple,

$$m + n \equiv LCM(n, m)$$

multiplication with the greatest common divisor,

$$m \cdot n \equiv GCD(n, m)$$

$m \subset n$ to mean "m is a factor of n",

$$m \subset n \equiv \left(n = 0 mod(m)\right)$$

and

$$n' \equiv 30/n$$

satisfies all laws of the set algebra.

2. Using definitions from the classwork handout, devise logical arguments proving each of the following properties of algebra and partial ordering operations on sets and draw Venn diagrams where possible.
   a. $\mathrm{A} \cdot (\mathrm{B} + \mathrm{C}) = \mathrm{A} \cdot \mathrm{B} + \mathrm{A} \cdot \mathrm{C}$
   b. $A + (B \cdot C) = (A + B) \cdot (A + C)$
   c. $(A \subset B) \Leftrightarrow A + B = B$
   d. $(A \subset B) \Leftrightarrow A \cdot B = A$
   e. $(A + B)' = A' \cdot B'$
   f. $(A \cdot B)' = A' + B'$
   g. $(A \subset B) \Leftrightarrow (B' \subset A')$
   h. $(A + B)' = A' \cdot B'$
   i. $(A' + B')' + (A' + B\,)' = A$
3. Rewrite the following properties of set algebra and partial ordering operations on sets in the form of logical propositions, following the first example.

a. $[\mathrm{A} \cdot (\mathrm{B}+\mathrm{C}) = \mathrm{A} \cdot \mathrm{B} + \mathrm{A} \cdot \mathrm{C}] \Leftrightarrow \left[(\mathrm{x} \in \mathrm{A}) \wedge \left((\mathrm{x} \in \mathrm{B}) \vee (\mathrm{x} \in \mathrm{C})\right)\right] = \left[\left((\mathrm{x} \in \mathrm{A}) \wedge (\mathrm{x} \in \mathrm{B})\right) \vee \left((\mathrm{x} \in \mathrm{A}) \wedge (\mathrm{x} \in \mathrm{C})\right)\right]$

b. $A + (B \cdot C) = (A + B) \cdot (A + C)$

c. $(A \subset B) \Leftrightarrow A + B = B$

d. $(A \subset B) \Leftrightarrow A \cdot B = A$

e. $(A + B)' = A' \cdot B'$

f. $(A \cdot B)' = A' + B'$

g. $(A \subset B) \Leftrightarrow (B' \subset A')$

h. $(A + B)' = A' \cdot B'$

i. $(A' + B')' + (A' + B\,)' = A$

4. Four digits 1, 2, 3, 4, are written down in random order. Find probability that at least one digit will occupy its ordered place? What is the probability for five digits? What is the probability for n digits?

5. Write the first few terms in the following sequence ($n \geq 1$),

$$n\ fractions \begin{cases} \dfrac{1}{1+\dfrac{1}{1+\dfrac{1}{1+\cdots}}} \\ \qquad \qquad \dots + \dfrac{1}{1+x} \end{cases} = ?$$

   a. Try guessing the general formula of this fraction for any n.

   b. Using mathematical induction, try proving the formula you guessed.

6. Can you prove that,

   a.

$$\frac{3+\sqrt{17}}{2} = 3 + \frac{2}{3+\frac{2}{3+\frac{2}{3+\cdots}}}\ ?$$

   b. $1 = 3 - \dfrac{2}{3-\dfrac{2}{3-\dfrac{2}{3-\cdots}}}\ ?$

   c.

$$\frac{4}{2+\frac{4}{2+\frac{4}{2+\cdots}}} = 1 + \frac{1}{4+\frac{1}{4+\frac{1}{4+\cdots}}}\ ?$$

   Find these numbers?