Math 7: Handout 18
Coordinate System. Equations of Line and Circle.

Coordinates

Today we gave a brief overview of coordinate geometry:

- After we choose an origin (usually denoted O) and two perpendicular axes, every point in the plane is described by a pair of numbers, its x and y coordinates. We will write (a, b) for point with x coordinate a and y-coordinate b.
- Distance between two points is given by

$$d((x_1, y_1), (x_2, y_2)) = \sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2}$$

- A general equation of non-vertical line is $y = mx + b$; the number m is called the *slope* of this line. It can also be defined as follows: if (x_0, y_0) and (x_1, y_1) are two points on this line, then $\frac{y_1 - y_0}{x_1 - x_0} = m$.
  Another common form of writing the equation of a line is $ax + by = c$.
- Equation of a circle with center at (x_0, y_0) and radius r is $(x - x_0)^2 + (y - y_0)^2 = r^2$.

Homework

1. Show that two lines are parallel if and only if they have the same slope.
2. a. Show that 90° counterclockwise rotation sends point $(2, 1)$ to point $(-1, 2)$. Where would it send point (x, y)?
   *b. Show that two lines are perpendicular if and only if their slopes are related by $m_1 = -1/m_2$.
3. Find the equation of a line going through point $(5, 7)$ and having slope 2.
4. Find the equation of a line through two points, $(3, 4)$ and $(5, 7)$.
5. What is the equation of a circle centered in $O(-3, 1)$ and of radius 2.
6. What are the coordinates of the center and the radius of the circle defined by $(x + 7)^2 + (y - 3)^2 = 2$?
7. Show that $(3, 5)$ is equidistant from $(-1, 2)$ and $(3, 0)$. (*Equidistant* means that the distances are the same)
8. Let $A = (3, 5)$, $B = (6, 1)$ be two of the vertices of a square $ABCD$ (the vertices are labeled A, B, C, D going counterclockwise). Find the coordinates of points C, D and of the center of the square. Find the area of this square.
9. Let C be the circle with center at $(0, 1)$ and radius 2, and l - the line with slope 1 going through the origin. Find the intersection points of the circle C and line l, and compute the distance between them.
*10. Prove the following formula for the distance from a point to the line: the distance from point $P = (u, v)$ to the line given by equation $ax + by = 0$ is

$$d = \frac{|au + bv|}{\sqrt{a^2 + b^2}}$$

11. Prove that the set of all points P satisfying the following equation

$$\text{distance from } P \text{ to the origin} = 2 \cdot (\text{distance from } P \text{ to } (0, 3))$$

is a circle. Find its radius and center.