Math 7: Handout 14
Vieta Formulas.

Vieta formulas

If an equation $p(x) = 0$ has root a (i.e., if $p(a) = 0$), then $p(x)$ is divisible by $(x - a)$, i.e. $p(x) = (x - a)q(x)$ for some polynomial $q(x)$. In particular, if $x_1; x_2$ are roots of quadratic equation $ax^2+bx+c = 0$, then $ax^2+bx+c = a(x-x_1)(x-x_2)$.

Therefore, if $a = 1$, then

$$\begin{aligned} x_1 + x_2 &= b \\ x_1x_2 &= c \end{aligned}$$

These formulas are called *Vieta Formulas*.

Homework

1. Let a and b be some numbers. Use the formulas discussed in previous classes to express the following expressions using only $(a + b) = x$ and $ab = y$.
   Example: Let's express $a^2 + b^2$ using only $a + b$ and ab. We know that $(a + b)^2 = a^2 + 2ab + b^2$. From here, we get:
   $$a^2 + b^2 = (a + b)^2 - 2 \times ab = x^2 - 2 \times y$$
   a. $(a - b)^2$
   b. $\frac{1}{a} + \frac{1}{b}$
   c. $a - b$
   d. $a^2 - b^2$
   e. $a^3 + b^3$ (Hint: first compute $(a + b)(a^2 + b^2)$)
2. Let x_1, x_2 be roots of the equation $x^2 + 5x - 7 = 0$. Find
   a. $x_1^2 + x_2^2$
   b. $(x_1 - x_2)^2$
   c. $\frac{1}{x_2} + \frac{1}{x_2}$
   d. $x_1^2 + x_2^3$
3. Solve the following equations:
   a. $x^2 - 5x + 6 = 0$
   b. $x^2 = 1 + x$
   c. $\sqrt{2x + 1} = x$
   d. $x + \frac{1}{x} = 3$
4. Solve the equation $x^4 - 3x^2 + 2 = 0$
5. a. Prove that for any $a > 0$, we have $a + \frac{1}{a} \geq 2$, with equality only when $a = 1$.
   b. Show that for any $a, b \geq 0$, one has $\frac{a+b}{2} \geq \sqrt{ab}$. (The left hand side is usually called the *arithmetic mean* of a, b; the right hand side is called the *geometric mean* of a, b.)