Math 7: Handout 11
Binomial Theorem. Binomial Probabilities.

Binomial Theorem

Binomial coefficients that appear in Pascal Triangle have an important application in algebra. The allow us to find an expansion of expressions such that $(a+b)^2, (a+b)^3, (a+b)^4, (a+b)^5, (a+b)^6$?

$$(a+b)^n = \binom{n}{0}a^n + \binom{n}{1}a^{n-1}b + \cdots + \binom{n}{k}a^{n-k}b^k + \cdots + \binom{n}{n}b^n$$

Notice that the coefficients correspond to a row in Pascal's Triangle. The coefficient of the term $a^{n-k}b^k$ is $\binom{n}{k}$.

Square of a Sum, Difference of Squares

Recall the following formulas. They are all special cases of Binomial Theorem!

- $(a+b)^2 = a^2 + 2ab + b^2$ (square of a sum)
- $(a-b)^2 = a^2 - 2ab + b^2$ (square of a difference)
- $a^2 - b^2 = (a+b)(a-b)$ (difference of squares)
- $(a+b)^3 = a^3 + 3a^2b + 3ab^2 + b^3$ (cube of a sum)
- $(a-b)^3 = a^3 - 3a^2b + 3ab^2 - b^3$ (cube of a difference)

Binomial Probabilities

These numbers are also useful in calculating probabilities. Imagine that we have some event that happens with probability p ("success") and does not happen with probability $q = 1 - p$ ("failure"). Then the probability of getting k successes in n trials is

$$P(k \text{ successes in } n \text{ trials }) = \binom{n}{k}p^k q^{n-k}, \text{ where}$$

- p — probability of success in one try;
- $q = 1 - p$ — probability of failure in one try;
- n — number of trials;
- k — number of successes;
- $n - k$ — number of failures.

Example: You roll a die 100 times. What is the probability of getting a 6 exactly 20 times?
Solution: Here we have: $n = 100$, $k = 20$, $p = 1/6$, $q = 5/6$. Then

$$P = \binom{100}{20} \cdot \left(\frac{1}{6}\right)^{20} \left(\frac{5}{6}\right)^{80}$$