Program

- Basics of logic. Knights and knaves. NOT, AND, OR, IF.
- Sets. Notation. Union, intersection, complement. Cardinality.
- Factorials and permutations.
- Ruler and compass constructions: midpoint, perpendicular, bisector.
- Coordinates. Equation of the line.
- Distance between two points on a coordinate plane. Equation of the circle.
- Arithmetic sequence. Geometric sequence. Formula for the general term. Formula for the sum.

Problems

1. On the island of knights and knaves, you meet two inhabitants: Sue and Zippy. Sue says that Zippy is a knave. Zippy says, "I and Sue are knights." So who is a knight and who is a knave?

2. On the island of Knights and Knaves, you meet three inhabitants:Bozo, Carl and Joe. Bozo says that Carl is a knave. Carl tells you, 'Of Joe and I, exactly one is a knight.' Joe claims, 'Bozo and I are different.'

3. On the island of Knights and Knaves, a traveler meets two inhabitants: Carl and Bill. Bill says: "Carl is a Knave". Carl says: "If Bill is a Knight, then I am a Knight, too."

4. Prove that
$$\text{NOT}(A \text{ AND } B) \text{ is the same as } (\text{NOT } A) \text{ OR} (\text{NOT } B)$$

5. Write the truth table for each of the following formulas. Are they equivalent (i.e., do they always give the same value)?

   a. $(A \text{ OR } B) \text{ AND} (A \text{ OR } C)$

   b. $A \text{ OR} (B \text{ AND } C)$.

6. If today is Thursday, then Jane's class has library day. If Jane's class has library day, then Jane will bring home new library books. Jane brought no new library books. Therefore,…

7. Let us take the usual deck of cards. As you know, there are 4 suits, hearts, diamonds, spades and clubs, 13 cards in each suit.
   Denote:
   H=set of all hearts cards
   Q=set of all queens
   R=set of all red cards
   Describe by formulas (such as $H \cap Q$) the following sets:
   all red queens
   all black cards
   all cards that are either hearts or a queen
   all cards other than red queens
   How many cards are there in each set?

8. Let
   A=set of all people who know French
   B=set of all people who know German
   C=set of all people who know Russian
   Describe in words the following sets:
   (a) $A \cap B$ (b)$A \cup (B \cap C)$ (c) $(A \cap B) \cup (A \cap C)$ (d) $C \cap \overline{A}$.

9. In a class of 25 students, 10 students know French, 5 students know Russian, and 12 know neither. How many students know both Russian and French?

10. Let $A = [1, 3] = \{x \mid 1 \leq x \leq 3\}$, $B = \{x \mid x \geq 2\}$, $C = \{x \mid x \leq 1.5\}$. Draw on the number line the following sets: $\overline{A}$, $\overline{B}, \overline{C}, A \cap B, A \cap C, A \cap (B \cup C), A \cap B \cap C$.

11. Show that for two sets A, B, we have $|A \cup B| = |A| + |B| - |A \cap B|$.

12. A group of 6 club members always dine at the same round table in the club; there are exactly 6 chairs at the table. They decided that each day, they want to seat in a different order. Can they keep this for a year? Two years?

13. In a computer game, a wizard is more powerful than an orc, so when a wizard fights an orc, he has 60% chance of winning. If a wizard fights one by one a group of 5 orcs, what are the chances that he will defeat them all?

14. In how many ways can one arrange 5 books on a shelf?

15. Show how to find a midpoint of an interval using ruler and compass.

16. Show how to construct a bisector using ruler and compass.

17. Draw all points on the plane for which one has $x = y + 1$.

18. Point M has coordinates $(5, 7)$.

   a. Find coordinates of the point M_1 obtained from M by reflection around the x-axis
   b. Find coordinates of the point M_2 obtained from M by reflection around the diagonal line.

19. Draw the graphs of the following functions:

   a. $2x + 3y = 1$
   b. $2x - 1 = y$
   c. $y = |x| - 2$

20. Find the distance between points $(2, 4)$ and $(3, 7)$.

21. Write the equation of the circle with center at $(1, 1)$ and radius 5.

22. What are the first 2 terms for the arithmetic sequence $a_1, a_2, -9, -2, 5, \ldots$?

23. In arithmetic sequence $a_{10} = 131$ and $d = 12$. What is a_1?

24. In arithmetic sequence $a_5 = 27$ and $a_{27} = 60$. Find the first term and the common difference.

25. Find the sum of the first 100 terms of the arithmetic sequence if $a_1 = 10$ and $a_{100} = 150$.

26. What are the first 2 terms for the geometric sequence $a_1, a_2, 24, 36, 54, \ldots$?

27. A geometric sequence has 99 terms, and the first term is 12 and the last term is 48. What is the 50th term?

28. Compute
$$\frac{1}{2} + \frac{1}{2^2} + \frac{1}{2^3} + \cdots + \frac{1}{2^{10}}$$

29. Find the infinite sum
$$1 + \frac{1}{3} + \frac{1}{9} + \frac{1}{27} + \ldots.$$