# MATH 7 <br> HOMEWORK 25: 

APRIL 14, 2019

## 1. Radians

We are introducing a new way to measure an angle.
A radian is the angle subtended at the center of a circle by an arc equal in length to the radius.

Remember that it doesn't matter how big or small a circle is, the angle at the center of a circle is always proportional to the arc subtended by the angle. The circumference of a circle $(2 \pi R)$ divided by the length of the radius is a constant $=2 \pi$. So $\frac{2 \pi R}{R}=2 \pi$ We say that the entire $360^{\circ}$ angle at the center of the circle has $2 \pi$ radians.
Half of it, $\pi$ radians is $180^{\circ}$, then 1 radian $=\frac{180}{\pi}=57.2958^{\circ}$ (approximately)
We can easily convert from radians to degrees and from degrees to radians. $180^{\circ}=\pi$ radians, $1^{\circ}=\frac{\pi}{180}$ radians, $\theta^{\circ}=\left(\theta \times \frac{\pi}{180}\right)$ radians
For example an angle of $90^{\circ}=\frac{\pi}{2}, 45^{\circ}=\frac{\pi}{4}, 60^{\circ}=\frac{\pi}{3}, 30^{\circ}=\frac{\pi}{6}$
Observe that we don't evaluate $\pi$, we carry it along.

## 2. Length and Area of a Sector of a Circle

A sector of a circle is a part of a circle in the shape of a pizza slice. If $a=$ length of an arc and $\theta=$ number of radians in the angle subtending the arc, then $\frac{\operatorname{arc}}{\text { radius }}=$ number of radians the arc subtends. So $\frac{a}{r}=\theta$ and $a=r \theta$
To find the area of a sector A, if $\theta_{1}$ is the angle in degrees $\frac{A}{\pi r^{2}}=\frac{\theta_{1}}{360}, A=\frac{\theta_{1} \pi r^{2}}{360^{\circ}}$, where $\theta$ (in radians) $=\frac{\theta_{1} \pi}{180}, A=\frac{\theta r^{2}}{2}, A=\frac{a r}{2}$

## 3. Problems

1. What is the number of degrees in each of the following angles: $\frac{\pi}{3}, \frac{\pi}{12}, \frac{3 \pi}{2}, \frac{2 \pi}{3}, \frac{3 \pi}{4}$
2. Express the following angles in radians, using fractions of $\pi$ : $15^{\circ}, 72^{\circ}, 66^{\circ}, 105^{\circ}$
3. A circular arc is 154 cm , the radius is 252 cm . What is the angle subtented at the center of the circle in radians and degrees?
4. The angle of a triangle are in the ratio $3: 4: 5$. Express them in radians.
5. In an arcade game, the monster is the sector of a circle of radius 1 cm . The missing piece (the mouth) has central angle $60^{\circ}$. What is the perimeter of the monster in

$\mathrm{cm}^{2}$ ?
6. A semicircle of diameter 1 sits on top of a semicircle of diameter 2 . The shaded area inside the smaller semicircle and outside the larger semicircle is called a lune. What

is the area of this lune?
7. What is the perimeter of the figure which consists of three congruent tangent circles of radius 6 cm ?


## 4. Review Problems

8. Review homeworks 1 to 4 about exponents and radicals.
9. Simplify $\left(2\left(x^{2}\right)^{-2}\right)^{2}$
10. Simplify: $\frac{\sqrt{13}}{\sqrt{56}} \times \frac{\sqrt{7}}{\sqrt{26}}$
11. Simplify: $\frac{1}{\sqrt{1}+\sqrt{2}}+\frac{1}{\sqrt{2}+\sqrt{3}}+\ldots+\frac{1}{\sqrt{49}+\sqrt{50}}$
