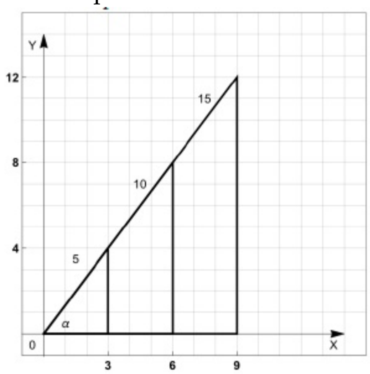
Math 7: Homework 24
Basic Trigonometry – 2.

Tangent $\tan(\alpha)$

Now we can also define the 3rd trigonometric ratio:

$$\tan(\alpha) = \frac{\sin(\alpha)}{\cos(\alpha)} = \frac{\text{opposite side/hypotenuse}}{\text{adjacent side/hypotenuse}} = \frac{\text{opposite side}}{\text{adjacent side}}$$

For example:



$$\sin\alpha = \frac{\text{opposite side}}{\text{hypotenuse}} = \frac{4}{5} = \frac{8}{10} = \frac{12}{15}$$

$$\cos\alpha = \frac{\text{adjacent side}}{\text{hypotenuse}} = \frac{3}{5} = \frac{6}{10} = \frac{9}{15}$$

$$\tan\alpha = \frac{\text{opposite side}}{\text{adjacent side}} = \frac{4}{3} = \frac{8}{6} = \frac{12}{8}$$

Trigonometric Functions							
Function	Notation	Definition	0°	30°	45°	60°	90°
sine	$\sin(\alpha)$	$\frac{\text{opposite side}}{\text{hypotenuse}}$	0	$\frac{1}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{3}}{2}$	1
cosine	$\cos(\alpha)$	$\frac{\text{adjacent side}}{\text{hypotenuse}}$	1	$\frac{\sqrt{3}}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{1}{2}$	0
tangent	$\tan(\alpha)$	$\frac{\text{opposite side}}{\text{adjacent side}}$	0	$\frac{1}{\sqrt{3}}$	1	$\sqrt{3}$	∞

Trigonometric Identities and Laws of Sines

The most prominent trigonometric identity is:

$$\sin^2(\alpha) + \cos^2(\alpha) = 1.$$

Let us try to derive it:

- A right triangle with hypotenuse c and an angle α is given. Express the remaining 2 sides (a and b) of triangle using only c and α.
- Using expressions obtained for a and b, express the hypotenuse c and simplify.

Law of Sines: Given a triangle ΔABC with sides a, b, and c, the following is always true:

$$\frac{a}{\sin(A)} = \frac{b}{\sin(B)} = \frac{c}{\sin(C)}.$$

Proof: To see why the Law of Sines is true, refer to the Figure 1. The height of the triangle $h = b\sin C$, and therefore the area of the triangle is $S = \frac{1}{2}ab\sin C$. Similarly, $S = \frac{1}{2}bc\sin A = \frac{1}{2}ac\sin B$. Thus,

$$bc\sin A = ac\sin B = ab\sin C$$

Dividing by abc, we get:

$$\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}.$$

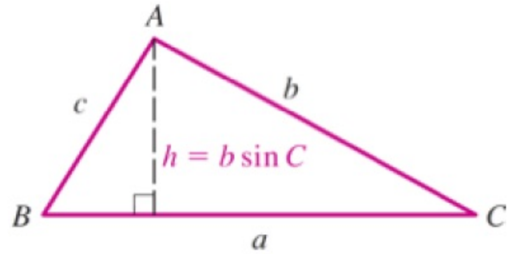


Figure 1. Law of Sines

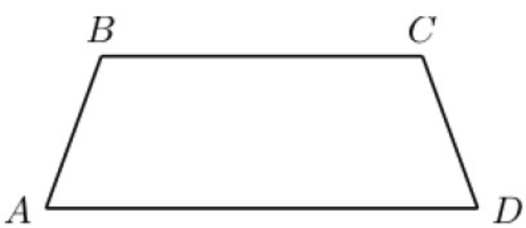
Laws of Cosines

The Law of Cosines is an extension to Pythagora's theorem and can be used for any type of triangle. If you know an angle and two sides, or if you know all three sides, you can compute the angles or the missing side of a triangle.

$$c^2 = a^2 + b^2 - 2ab \cdot cos(C)$$

Homework

1. If a right triangle ΔABC has sides $AB = 3 * \sqrt{3}$ and $BC = 9$, and side AC is the hypotenuse, find all 3 angles of the triangle.
2. The area of a right triangle is 36 square meters. The legs of the triangle have the ratio of $2 : 9$. Find the hypotenuse of the triangle.
3. In a triangle ΔABC, we have $\angle A = 40°$, $\angle B = 60°$, and $AB = 2$ cm. What is the remaining angle and side lengths? (Hint: Use Law of sines)
4. In an isosceles triangle, the angle between the equal sides is equal to $30°$, and the height is 8. Find the sides of the triangle.
5. A right triangle ΔABC is positioned such that A is at the origin, B is in the 1st quadrant ($B_x > 0$ and $B_y > 0$) and C is on the positive horizontal axis ($C_x > 0$ and $C_y = 0$). If length of side AB is 1, and AB makes a $35°$ angle with positive x axis, what are the coordinates of B?
6. Consider a parallelogram $ABCD$ with $AB = 10$, $AD = 4$ and $\angle BAD = 50°$. Find the length of diagonal AC.
7. A regular heptagon (7 sides) is inscribed into a circle of radius 1.
   a. What is the perimeter of the heptagon?
   b. What is the area of the heptagon?
8. In the trapezoid below, $AD = 5$ cm, $AB = 2$ cm, and $\angle A = \angle D = 70°$. Find the length BC and the diagonals. [You can use: $\sin(70°) \approx 0.94$, $\cos(70°) \approx 0.34$.]



9. Two straight roads, Elm Street and Pine Street, intersect creating a 40° angle, as shown in the accompanying diagram. John's house (J) is on Elm Street and is 3.2 miles from the point of intersection. Mary's house (M) is on Pine Street and is 5.6 miles from the intersection. Find, to the nearest tenth of a mile, the direct distance between the two houses.