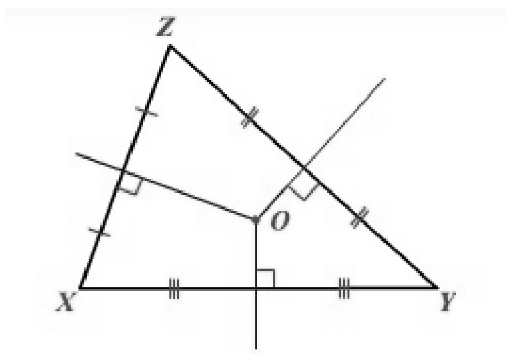
MATH 7: HOMEWORK 20

MARCH 3, 2019

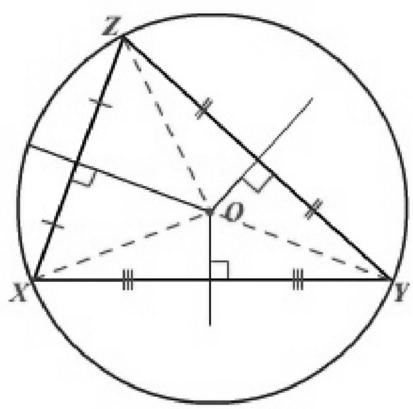
1. Perpendicular Bisectors in a Triangle

We have seen in a previous homework how to construct the perpendicular bisector given 2 points, using only a ruler and compass.
A perpendicular bisector is the locus of all points equally away from two given points.
A perpendicular bisector in a triangle is perpendicular and bisects the side of a triangle.

Theorem 1. *In a $\triangle ABC$, all 3 perpendicular bisectors intersect in one point only. We say they are* **concurrent**. *This point is called the circumcenter of a triangle*



Theorem 2. *The intersection point of all 3 perpendicular bisectors in a triangle is the center of the circumcircle of the triangle. The circumcircle is the circle that inscribes the triangle.*

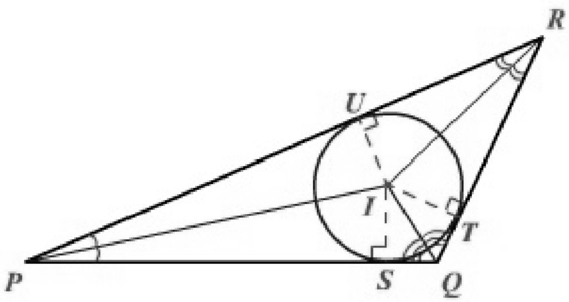


2. Angle bisectors in a triangle

We have seen how to build an angle bisector using only ruler and compass. **An angle bisector is the locus of all points equally away from two lines.**

Theorem 3. *All three angle bisectors in a triangle are concurrent.*

Theorem 4. *The point where all three angle bisectors meet is called the incenter. It is the center of the circle that is inscribed in the triangle*



3. Altitudes in a triangle

An altitude in a triangle is the line through a vertex of the triangle and perpendicular on the opposite side.

Theorem 5. *All three altitudes in a triangle are concurrent and the intersection point is called the orthocenter.*

4. Medians in a triangle

A median in a triangle connects a vertex with the middle of the opposite side of a triangle.

Theorem 6. *All three medians in a triangle are concurrent. The intersection point is called centroid or center of mass.*

Homework

1. Solve problems 9-12 on page 150 in the E-Z Geometry book.

2. Prove Theorems 1,2,3,4

3. Prove Theorem 5. For a given triangle $\triangle ABC$, draw through each vertex a line parallel to the opposite side of the triangle. This will produce a larger triangle; denote it $\triangle A'B'C'$.
   (a) Show that triangles $\triangle ABC$, $\triangle A'B'C'$ are similar and find the coefficient.
   (b) Show that altitudes of $\triangle ABC$ are perpendicular bisectors of $\triangle A'B'C'$.
   (c) Show that the three altitudes of $\triangle ABC$ intersect at a single point.

4. Prove Theorem 6. Let AM, BN be the medians of a triangle $\triangle ABC$, and let O be their intersection point.
   (a) Show that triangles $\triangle AOB$ and $\triangle MON$ are similar and find the coefficient.
   (b) Show that $AO = 2OM$.
   (c) Let O' be the intersection point of AM with the third median, CK. Show that $O = O'$ and thus, all three medians intersect at a single point.