FEB 10, 2019

1. Similar triangles

We say that triangles $\triangle ABC$ and $\triangle A'B'C'$ are similar with coefficient k if $m \angle A = m \angle A', m \angle B = m \angle B'$, $\angle C = \angle C'$ and

$$\frac{A'B'}{AB} = \frac{B'C'}{BC} = \frac{A'C'}{AC} = k.$$

We will use notation $\triangle ABC \sim \triangle A'B'C'$.

Theorem Consider a triangle $\triangle ABC$ and let $B' \in \overrightarrow{AB}$, $C' \in \overrightarrow{AC}$ be such that lines $\stackrel{\longleftrightarrow}{BC}$ and $\stackrel{\longleftrightarrow}{B'C'}$ are parallel. Then $\triangle ABC \sim$ $\triangle A'B'C'.$



Theorem [Similarity via AA] Let $\triangle ABC$, $\triangle A'B'C'$ be such that $m \angle A = m \angle A'$, $m \angle B = m \angle B'$. Then these triangles are similar.

Proof Let $k = \frac{A'B'}{AB}$. Construct a triangle $\triangle A''B''C''$ which is similar to $\triangle ABC$ with coefficient k. Then A'B' = A''B'', and $m \angle A = m \angle A' = m \angle A''$, $m \angle B = m \angle B' = m \angle B''$. Thus, by ASA, $\triangle A'B'C' \cong$ $\triangle A''B''C''.$

Theorem [Similarity via SAS] Let $\triangle ABC$, $\triangle A'B'C'$ be such that $\angle A = \angle A'$, $\frac{A'B'}{AB} = \frac{A'C'}{AC}$. Then these triangles are similar.

Theorem [Similarity via SSS] Let $\triangle ABC$, $\triangle A'B'C'$ be such that

$$\frac{A'B'}{AB} = \frac{A'C'}{AC} = \frac{B'C'}{BC}$$

Then these triangles are similar.

One of the most important applications of the theory of similar triangles is to the study of right triangles and Pythagorean theorem.

A right triangle is a triangle in which one of the angles is a right angle. A hypotenuse is the side opposing the right angle; two other sides are called legs.

Theorem Let $\triangle ABC$ be a right triangle, with $\angle C$ being the right angle. Let CM be the altitude of angle C. Then triangles $\triangle ABC$, $\triangle ACM$, $\triangle CBM$ are all similar.

Proof It immediately follows from AA similarity rule.

This theorem immediately implies a number of important relations between various lengths in these triangles. We will give one of them. Denote for brevity a = BC, b = AC, c = AB, x = AM, y = MB, h = CM. Then we have x : h = b : a, y : h = a : b, so

$$\frac{x}{h} \times \frac{y}{h} = 1$$

or $xy = h^2$.



C

HOMEWORK

In the problems about constructing something with a ruler and compass, the ruler can only be used for drawing straight lines through two given points; you can not use it to measure distances. As before, you can freely use previous results and constructions instead of repeating all the steps.

- 1. Problems 8,9,12,17,18,19,20 starting on page 226 in the textbook.
- **2.** Let ABCD be a trapezoid with bases AD = 9, BC = 6, such that the height (distance between the bases) is equal to 5. Let O be the intersection point of lines AB, CD.
 - (a) Show that triangles $\triangle OBC$, $\triangle OAD$ are similar and find the coefficient.
 - (b) Find the distance from O to AD (i.e., length of the perpendicular).
- **3.** Given three line segments, of lenghts 1, x, y, construct the line segments of lenghts xy, x/y, using only ruler and compass.
- 4. In a triangle $\triangle ABC$, let D be midpoint of side BC, E midpoint of side AC, F midpoint of side AB. Prove that $\triangle DEF$ is similar to triangle $\triangle ABC$ with coefficient 1/2.
- 5. Use the following figure to prove that an angle bisector in a triangle $\triangle ABC$ divides the opposite side in the same proportion as the two

S

R

6. In triangle PQR, PQ, PR are the same. Point S lies on QR so that QS = PS and $\langle RPS = 75^{\circ}$. What is the size of $\langle QRP$? (Math Kangaroo)

adjoining sides: $\frac{x}{y} = \frac{\hat{B}A}{BC}$.