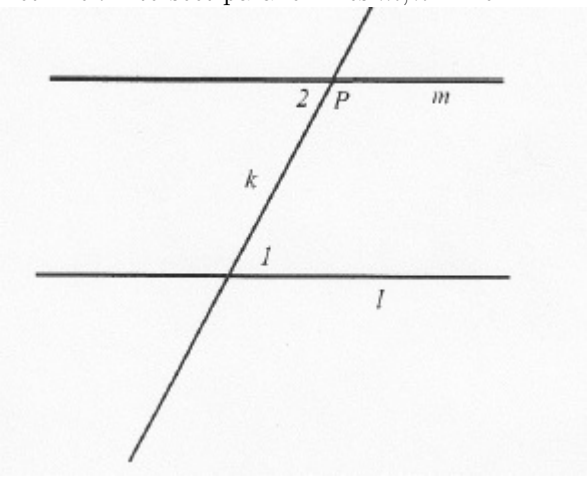
MATH 7: HOMEWORK 16

JAN 27, 2019

1. Alternate Interior Angles

Let line l intersect parallel lines m, n. Then $\angle 1 = \angle 2$ are as shown in the figure below.



The converse is also true and gives us a way to prove that two lines are parallel. If line l intersects lines m, n and $\angle 1 = \angle 2$ then m, n are parallel.

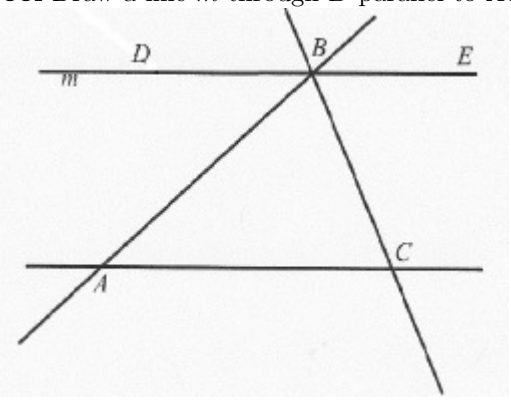
2. Sum of angles of a triangle

Definition A triangle is a figure consisting of three distinct points A, B, C (called vertices) and line segments $\overline{AB}$, $\overline{BC}$, $\overline{AC}$. We denote such a triangle by $\triangle ABC$.

Similarly, a quadrilateral is a figure consisting of 4 distinct points A, B, C, D and line segments $\overline{AB}$, $\overline{BC}$, $\overline{CD}$, $\overline{DA}$ such that these segments do not intersect except at A, B, C, D.

Theorem The sum of measures of angles of a triangle is $180°$.

Proof Draw a line m through B parallel to $\overleftrightarrow{AC}$. Let D, E be points on m as shown in the figure below.



Then $m\angle DBA = m\angle A$ as alternate interior angles, $m\angle CBE = m\angle C$. On the other hand, we have

$$m\angle DBA + m\angle B + m\angle CBE = 180°$$

Thus, $m\angle A + m\angle B + m\angle C = 180°$.

Theorem For a triangle $\triangle ABC$, let D be a point on continuation of side AC, so that C is between A and D. Then $m\angle CBD = m\angle A + m\angle B$. (Such an angle is called the exterior angle of triangle ABC.)

Theorem Sum of angles of a quadrilateral is equal to $360°$.

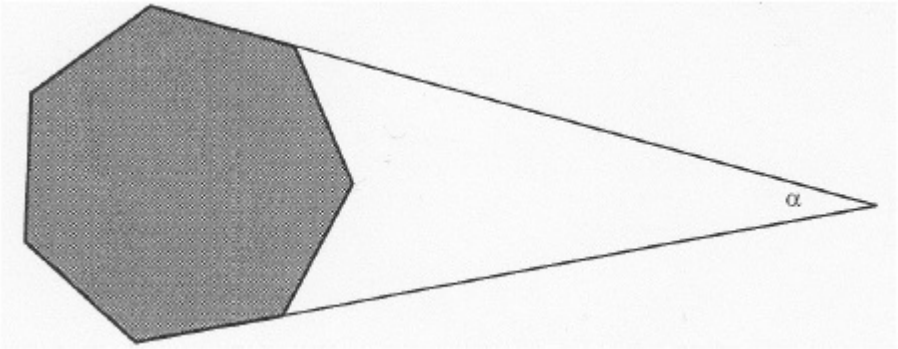
Homework

1. Exercises 3,4,5,7 on pages 76-77 in the book. [Notation $\angle 1 \cong \angle 2$ means $m\angle 1 = m\angle 2$.]

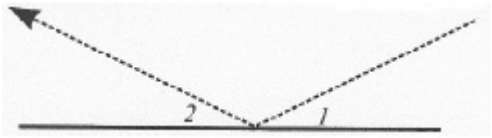
2. Exercises 6,7,8 on page 98 in the book.

3. Deduce a formula for the sum of angles in a polygon with n vertices.

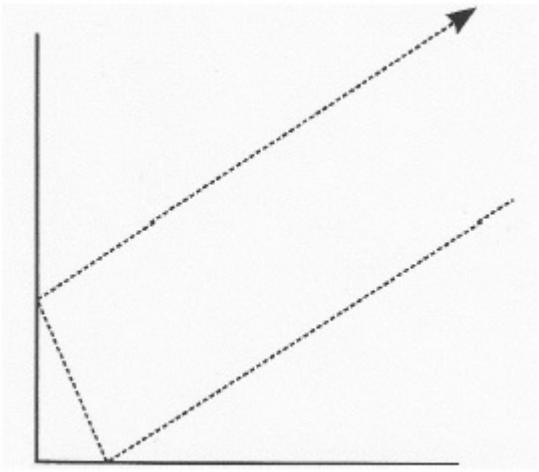
4. In the figure below, all angles of the 7-gon are equal. What is angle α?[By the way: α is a Greek letter, pronounced "alpha"; mathematicians commonly use Greek letters to denote angles]



5. The reflection law states that the angles formed by the incoming light ray and the reflected one with the surface of the mirror are equal: $m\angle 1 = m\angle 2$



Using this law, show that a corner made of two perpendicular mirrors will reflect any light ray exactly back: the reflected ray is parallel to the incoming one:



This property – or rather, similar property of corners in 3-D – is widely used: reflecting road signs, tail lights of a car, reflecting strips on clothing are all contrsructed out of many small reflecting corners so that they reflect the light of a car headlamp exactly back to the car.

6. Show that if, in a quadrilateral $ABCD$, diagonally opposite angles are equal ($m\angle A = m\angle C$, $m\angle B = m\angle D$), then opposite sides are parallel. [Hint: show first that $m\angle A + m\angle B = 180°$.]