MATH 7
HOMEWORK 14: COORDINATE PLANE. EQUATION OF A LINE AND A CIRCLE
JAN 13, 2019

COORDINATES

Today we gave a brief overview of coordinate geometry:
- After we choose an origin (usually denoted O) and two perpendicular axes, every point in the plane is described by a pair of numbers, its x and y coordinates. We will write (a, b) for point with x coordinate a and y-coordinate b.
- Distance between two points is given by
$$d((x_1, y_1), (x_2, y_2)) = \sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2}$$
- A general equation of non-vertical line is $y = mx + b$; the number m is called the *slope* of this line. It can also be defined as follows: if (x_0, y_0) and (x_1, y_1) are two points on this line, then $\frac{y_1 - y_0}{x_1 - x_0} = m$.
  Another common form of writing the equation of a line is $ax + by = c$.
- Equation of a circle with center at (x_0, y_0) and radius r is $(x - x_0)^2 + (y - y_0)^2 = r^2$.

HOMEWORK

1. Show that two lines are parallel if and only if they have the same slope.

2. (a) Show that $90°$ counterclockwise rotation sends point $(2, 1)$ to point $(-1, 2)$. Where would it send point (x, y)?

*(b) Show that two lines are perpendicular if and only if their slopes are related by $m_1 = -1/m_2$.

3. Find the equation of a line going through point $(5, 7)$ and having slope 2.

4. Find the equation of a line through two points, $(3, 4)$ and $(5, 7)$.

5. Let $A = (3, 5)$, $B = (6, 1)$ be two of the vertices of a square $ABCD$ (the vertices are labeled A, B, C, D going counterclockwise). Find the coordinates of points C, D and of the center of the square. Find the area of this square.

6. Let C be the circle with center at $(0, 1)$ and radius 2, and l - the line with slope 1 going through the origin. Find the intersection points of the circle C and line l, and compute the distance between them.

***7.** Prove the following formula for the distance from a point to the line: the distance from point $P = (u, v)$ to the line given by equation $ax + by = 0$ is
$$d = \frac{|au + bv|}{\sqrt{a^2 + b^2}}$$

8. Prove that the set of all points P satisfying the following equation

distance from P to the origin $= 2 \cdot$ (distance from P to $(0, 3)$)

is a circle. Find its radius and center.