HOMEWORK 6 MATH 7B

OCT. 28, 2018

Geometric Sequences

A sequence of numbers is a geometric sequence if the next number in the sequence is the current number times a constant called the **common ratio**, let's name it q.
Consider the sequence: 6,12, 24, 48,... The first term in the sequence is $b_1 = 6$, the second is $b_2 = 6 \times 2 = 12$ and so on.
The common ratio is $q = 2$. Indeed $b_3 = b_2 \times q = 12 \times 2 = 24$ and $b_4 = 24 \times 2 = 48$.
What is the n^{th} term? For example what is b_{10}?
$b_1 = 6$
$b_2 = b_1 \times q = 6 \times 2 = 12$
$b_3 = b_2 \times q = (b_1 \times q) \times q = b_1 \times q^2 = 6 \times 2^2 = 24$
$b_4 = b_3 \times q = (b_1 \times q^2) \times q = b_1 \times q^3 = 6 \times 2^3 = 48$
...
$\mathbf{b_n = b_1 \times q^{n-1}}$
So $b_{10} = b_1 \times q^9 = 6 \times 2^9 = 6 \times 512 = 3072$

Properties Of Geometric Sequence

Any term of a geometric sequence is the geometric mean of its neighbors.

$$\mathbf{b_n = \sqrt{b_{n-1} \times b_{n+1}}}$$

Sum of A Geometric Sequence

$$\mathbf{S = b_1 + b_2 + ...b_n = \frac{b_1 \times (1 - q^n)}{1 - q}}$$

To prove this, we write the sum and multiply it by q.
$S = b_1 + b_2 + ... + b_{n-1} + b_n$
$qS = qb_1 + qb_2 + ... + qb_{n-1} + qb_n$
where the last term can be written also as $qb_n = q \times (b_1 \times q^{n-1}) = b_1 \times q^n$.
Subtract $qS - S = qb_n - b_1 = b_1 \times q^n - b_1 = b_1(q^n - 1)$
Factor S on the left hand side and divide by $q - 1$, $S = \frac{b_1 \times (q^n - 1)}{q - 1} = \frac{b_1 \times (1 - q^n)}{1 - q}$

Infinite Sum

If $q < 1$, we can calculate the sum of an infinite geometric sequence using the above formula. q^n will be 0, so $\mathbf{S = \frac{b_1}{1 - q}}$

Homework

1. Write the first 5 terms of a geometric progression if $b_1 = -20$ and $q = \frac{1}{2}$.

2. What are the first 2 terms of the geometric progression: $b_1, b_2, 24, 36, 54, ...$?

3. What is the common ratio of the geometric progression: $\frac{1}{2}, -\frac{1}{2}, \frac{1}{2}, ...$? What is b_{10}? What is b_{100}?

4. Add: $\frac{1}{2} + \frac{1}{2^2} + \frac{1}{2^3} + ... + \frac{1}{2^{10}}$

5. What is $1 - 2 + 2^2 - 2^3 + 2^4 - 2^5 + ... - 2^{15}$?

6. What is: $1 + x + x^2 + x^3 + ... + x^{100}$

7. A geometric progression has 99 terms, the first term is 12 and the last term is 48. What is the 50^{th} term?

8. If we put one grain of wheat on the first square of a chessboard, two on the second, then four, eight,..., approximately how many grains of wheat will there be? (you can use $2^{10} = 1024 \approx 10^3$).
Can you estimate the total volume of all this wheat and compare with the annual wheat harvest of the US, which is about 2 billion bushels. (A grain of wheat is about 10mm^3; a bushel is about 35 liters, or 0.035m^3)

9. How many multiples of 7 are there between 1 and 1000? Can you find the sum of them all?

10. Find the sum $1 + 3 + 5 + ... + 999$.