HOMEWORK 5 MATH 7B OCT. 21, 2018

ARITHMETIC SEQUENCES

A sequence of numbers is an arithmetic sequence if the difference between consecutive terms is the same number, the **common difference** or **d**. For example, the sequence 1, 5, 9, 13, 17, ... is an arithmetic sequence because the difference between consecutive terms is d = 4. What is the n^{th} term? What is a_{100} in the example above? $a_1 = 1$ $a_2 = a_1 + d = 1 + 4 = 5$ $a_3 = a_2 + d = (a_1 + d) + d = a_1 + 2d = (1 + 4) + 4 = 1 + 2 \times 4 = 9$ $a_4 = a_3 + d = (a_2 + d) + d = ((a_1 + d) + d) + d = a_1 + 3d = 1 + 3 \times 4 = 13$

The pattern is $\mathbf{a_n} = \mathbf{a_1} + (\mathbf{n} - \mathbf{1})\mathbf{d}$ So $a_{100} = a_1 + 99d = 1 + 99 \times 4 = 397$

PROPERTIES OF AN ARITHMETIC SEQUENCE

A useful property of an arithmetic sequence is that any term is the arithmetic mean of its neighbors.

$$a_n = \frac{a_{n-1} + a_{n+1}}{2}$$

Proof:

$$a_n = a_{n-1} + d$$
$$a_n = a_{n+1} - d$$

Add these and $2a_n = a_{n-1} + a_{n+1}$ and divide by 2.

Another property of arithmetic sequences is that the common difference between any two terms a_s and a_t is

$$d = \frac{a_s - a_t}{s - t}$$

SUM OF AN ARITHMETIC SEQUENCE

$$S = a_1 + a_2 + a_3 + \dots + a_n = n \times \frac{a_1 + a_n}{2}$$

To prove this, we write the sum in 2 ways, in increasing and decreasing order:

$$S = a_1 + a_2 + \dots + a_n$$

$$S = a_n + a_{n-1} + \dots + a_1$$

Adding and noticing that $a_1 + a_n = a_2 + a_{n-1} = a_3 + a_{n-2} = ...$ $2S = (a_1 + a_n) \times n$ and divide by 2

$$S = n \times \frac{a_1 + a_n}{2}$$

Homework

- **1.** Write the first 5 terms of an arithmetic sequence if $a_1 = 7$ and d = 2
- **2.** What are the first 2 terms for the sequence $a_1, a_2, -9, -2, 5, \dots$
- **3.** $a_{10} = 131$ and d = 12. What is a_1 ?
- 4. $a_5 = 27$ and $a_{27} = 60$. Find the first term and the common difference. 5. Find the common difference in an arithmetic sequence if the 9th term is 18 and the 11^{th} term is 44.
- 6. Find the sum of the first 100 terms if $a_1 = 10$ and $a_{100} = 150$.
- 7. Find the sum of all odd numbers from 1 to 2019.
- 8. Continue the following sequence: $-2, 1, 6, 13, 22, \dots$ (Hint: look at the differences of successive terms)
- **9.** A right triangle has sides of 5,12,13. What is radius of the inscribed semicircle? (Math Kangaroo)



Hint: Express the area of the right triangle in 2 ways using the radius of the semicircle. 10. M is the midpoint of AD, and MN is perpendicular to AC. What is the ratio of the shaded area to the total area of the square? (Math Kangaroo)

