

Math 6 a/b: Homework 22
Homework #22 is due April 7.

Arithmetic sequences

A sequence of numbers is an *arithmetic sequence* if the difference between consecutive terms is the same number, a common difference, let's call it d . For example, consider the sequence:

$$1, 5, 9, 13, 17, \dots$$

The first term in the sequence is: $a_1 = 1$, the second is $a_2 = 5$, and so on. The difference is $d = 4$. What is the n th term? For example, what is a_{100}

$$a_1 = 1$$

$$a_2 = a_1 + d = 1 + 4 = 5$$

$$a_3 = a_2 + d = a_1 + 2d = 1 + 2 \times 4 = 9$$

$$a_4 = a_3 + d = a_1 + 3d = 1 + 3 \times 4 = 13$$

...

$$a_n = a_1 + (n - 1)d$$

$$\text{So } a_{100} = a_1 + 99d = 1 + 99 \times 4 = 397$$

Property of an arithmetic sequence

A property of an arithmetic sequence is that any term is the arithmetic mean of its neighbors.

$$a_n = \frac{a_{n-1} + a_{n+1}}{2}$$

To prove this we can write:

$$a_n = a_{n-1} + d$$

$$a_n = a_{n+1} - d$$

Add them up and we have:

$$2a_n = (a_{n-1} + d) + (a_{n+1} - d)$$

$$2a_n = a_{n-1} + a_{n+1}$$

Dividing by 2:

$$a_n = \frac{a_{n-1} + a_{n+1}}{2}$$

To find the common difference between 2 terms a_s and a_t

$$d = \frac{a_s - a_t}{s - t}$$

Sum of an arithmetic sequence

$$S = a_1 + a_2 + a_3 + \cdots + a_n = n \times \frac{a_1 + a_n}{2}$$

To prove this, we write the sum in 2 ways, in increasing order and in decreasing order:

$$S = a_1 + a_2 + a_3 + \cdots + a_n$$

$$S = a_n + a_{n-1} + a_{n-2} + \cdots + a_1$$

Adding up:

$$2S = (a_1 + a_n) + (a_2 + a_{n-1}) + (a_3 + a_{n-2}) + \cdots$$

We notice that:

$$a_1 + a_n = a_2 + a_{n-1} = a_3 + a_{n-2} = \cdots$$

$$2S = (a_1 + a_n) \times n$$

$$S = \frac{(a_1 + a_n) \times n}{2}$$

Homework Problems

1. Write the first 5 terms of an arithmetic sequence if $a_3 = 7$ and $d = 12$
2. What are the first 2 terms for the sequence $a_1, a_2, -9, -2, 5, \dots$
3. $a_{10} = 131$ and $d = 12$, what is a_1 ?
4. $a_5 = 27$ and $a_{27} = 60$. Find the first term and the common difference.
5. Find the common difference in an arithmetic sequence if the 9th term is 18 and the 11th term is 44.
6. Find the sum of the first 100 terms if $a_1 = 10$ and $a_{100} = 150$
7. Find the sum of all odd numbers from 1 to 2011.
8. Can you continue the following sequence: $-2, 1, 6, 13, 22, \dots$
[Hint: look at the differences of successive terms!]
9. Find the shortest distance from the origin $(0, 0)$ to a line given by the equation $y = -2x + 8$.
10. Compute the area of the rectangle $ABCD$ if A is at $(0, 0)$, B at $(2, 3)$, and D at $(-6, 8)$.
[It can be done in more than one way.]