Class notes: Constructions with a ruler and a compass

For the next couple of classes, we will be mostly interested in doing the geometric constructions with a ruler and a compass. Note that the ruler can only be used for drawing straight lines through two points, not for measuring distances!

When doing these problems, we need to:

- Give a recipe for constructing the required figure using only ruler and compass
- Explain why our recipe does give the correct answer

For the first part, our recipe can use only the following operations:

- Draw a line through two given points
- Draw a circle with center at a given point and given radius
- Find and label on the figure intersection points of already constructed lines and circles.

For the second part, we will frequently use the results below.

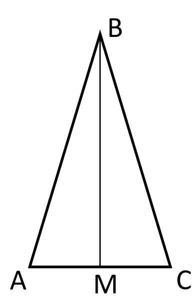
Congruence tests for triangles

Recall that, by definition, to check that two triangles are congruent, we need to check that corresponding angles are equal and corresponding sides are equal; thus, we need to check 6 equalities. However, it turns out that in fact, we can do this with fewer checks.

Axiom 1 (SSS Rule). If AB = A'B', BC = B'C' and AC = A'C' then $\Delta ABC \cong \Delta A'B'C'$.
Axiom 2 (Angle-Side-Angle Rule). If $\angle A = \angle A'$, $\angle B = \angle B'$ and AB = A'B', then $\Delta ABC \cong \Delta A'B'C'$.
This rule is commonly referred to as the ASA rule.
Axiom 3 (SAS Rule). If AB = A'B', AC = A'C' and $\angle A = \angle A'$, then $\Delta ABC \cong \Delta A'B'C'$.

Isosceles triangle

Recall that the triangle ΔABC is called isosceles if AB = BC.

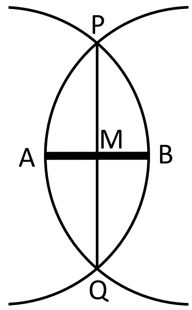


Theorem.
1. *In an isosceles triangle, base angles are equal: $\angle A = \angle C$.*
2. *In an isosceles triangle, let M be the midpoint of the base AC. Then line BM is also the bisector of angle B and the altitude: BM is perpendicular to AC.*

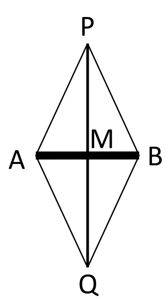
Example: finding the mid-point of the line segment

Problem: given two points A,B, construct the midpoint M of the segment AB.
Construction:

1. Draw a circle with center at A and radius AB
2. Draw a circle with center at B and radius AB
3. Mark the two intersection points of these circles by P,Q
4. Draw line through points P,Q
5. Mark the intersection point of line PQ with line AB by M. This is the midpoint.



Analysis:
This is a two-step argument. In this figure, triangles ΔAPQ and ΔBPQ are congruent (why?), so the corresponding angles are equal:



From this, we can see that ΔAPM ≅ ΔBPM, so AM = BM.

Math 6a/b: Homework 7

Homework #7 is <u>*due November 11*</u>. Please, write clearly which problem you are solving and show *all steps* of your solution. When drawing, use enough space and label all lines and points clearly.

1. Repeat the construction of the example problem on the previous page:
   Given two points A and B, construct the midpoint M of the segment AB.
   Prove (or explain) why in the construction above, the line PQ will in fact be a perpendicular to AB.

2. Given a segment with length *a*, construct an equilateral triangle with side *a (a = 5 cm).*
   Hint: Start by drawing a line segment on the page with a ruler without actually measuring its length. Think of this length as length ***a.*** *Remember, you are only allowed to "measure" length with your compass.*
3. Given a segment with length *a*, construct a regular hexagon with side *a*.
4. Given three segments with lengths *a, b, c,* construct a triangle with sides *a, b, c.*
5. Construct an isosceles triangle, given a base *b* and height *h*.

6. In the figure, ABCD is a rectangle, and M is the midpoint of BC. Prove that the triangle AMD is isosceles.