Math 6a/b: Homework 19
Homework #19 is due March 10.

Factorials and permutations

If we are to choose *k* objects from a collection of *n* so that a) order matters and b) no repetitions are allowed, then there are

$$n(n-1)\dots(n-(k-1)) \quad (k \text{ factors})$$

ways to do it.

If we take $k = n$, it means that we are selecting (one-by-one) all *n* objects, which gives the number of possible ways to order *n* objects:

$$n! = n(n-1)(n-2)\dots(2)(1)$$

n! is read as 'n factorial'.

For example: there are 52! ways to mix cards from a regular deck of cards.
Note that the number *n!* grows very fast: 2! = 2, 3! = 6, 4! = 24, 5! = 120, 6! = 620, and note that we define 0! = 1.

Homework

In all the problems that ask you to compute something, it is sufficient to write an expression for the answer, e.g., $1/2^{11}$, it is not necessary to perform the multiplication.

1. About 1/6 of Americans have blue eyes. If we choose 10 people at random, what is the probability that all of them have blue eyes? that none have blue eyes? that at least one has blue eyes?

2. A group of 6 club members always dine at the same table in the club; there are exactly 6 chairs at the table. They decided that each day, they want to seat in a different order. Can they keep this for a year? Two years?

3. How many ways are there to seat 15 students in a classroom which has 15 chairs? If the room has 25 chairs?

4. A puzzle consists of 9 small square pieces which must be put together to form a 3×3 square so that the pattern matches (this kind of puzzles is quite hard to solve!). It is known that there is only one correct solution. If you started trying all possible combinations at random, doing one new combination ***one second***, how long will it take you to try them all?

5. At a fair, you are offered to play the following game: you are tossing small balls in a large crate full of empty bottles; if at least one of the balls lands inside a bottle, you win a stuffed toy (worth about $5). Unfortunately, it is impossible to aim, so the game is just a matter of luck (or probability theory): every ball you toss has a 20% probability of landing inside the bottle.
(a) If you are given three balls, what is the probability that all three will be hits? That all three will be misses? That at least one will be a hit?
(b) Same questions for five balls.

6. (a) What is the probability that if we roll 2 dice, the sum will be at most 7?
(b) A and B are playing the following game. They roll 2 dice; if the sum is at most 7, A wins, and B pays him $1. Otherwise A loses and he pays to B $1. Would you prefer to play for A or for B in this game?

7. (a) What is the probability that if we roll 3 dice, all the numbers will be different?
(b) A and B are playing the following game. They roll 3 dice; if all numbers are different, A wins, and B pays him $2. Otherwise A loses and he pays to B $3. Would you prefer to play for A or for B in this game?

8. (a) In a class of 25 students, everyone chooses a date (e.g., March 13). How many combinations are possible? (Students only choose month and day, not year; February 29th is not allowed, so there are 365 different possibilities. Also, it matters who had chosen which day: combination where Bill has chosen March 12 and John, June 15 is considered different from the one where Bill has chosen June 15 and John March 12.)
(b) In the same situation, how many such combinations are possible if we additionally require that all dates must be different? *(c) Suppose now that each of these 25 students has chosen a date at random, not knowing the choices of others. What is the probability that all of these dates will be different? That at least 2 will coincide?