Math 6b/c: Homework 18
Homework #18 is due March 3.

Basic Probability

Basic probability rule, assuming all outcomes are equally likely:

$$P(win) = \frac{number\ of\ winning\ outcomes}{total\ number\ of\ possible\ outcomes}$$

Example: Probability of drawing a spade card out of the standard deck is $P(spade) = \frac{13}{52} = \frac{1}{4}$

Complement Rule

If probability of some event is P then the probability that this event will **not** happen is $1 - P$. For example, if we draw a card from the deck then the probability that it is not a spade is $1 - \frac{1}{4} = \frac{3}{4}$.

Product Rule

If we do two trials (e.g., rolling a die twice), then the probability of getting result A in the first trial and result B is the second one is

$$P(A, then\ B) = P(A)P(B)$$

if the result of the second trial <u>does not depend</u> on the results of the first one.

EXAMPLE: TOSSING A COIN

Question. If you toss a coin 10 times, what is the probability that all will be heads?

> **Answer**. $\left(\frac{1}{2}\right)^{10} = \frac{1}{2^{10}}$ (using a calculator, one can compute that this is $1/1024 \approx 0.001$, or 1/10th of 1%).

Question: If toss a coin 10 times, what is the probability that all will be tails?

> **Answer**. The same as above.

Question: If we toss a coin 10 times, what is the probability that *at least one* will be heads?

> **Answer**: Unfortunately, there are very many combinations which give at least one heads. In fact, it is easier to say which combinations do not give at least one heads: there is exactly one such combination, all tails. The probability of obtaining this combination is, as we computed, $\left(\frac{1}{2}\right)^{10} = \frac{1}{1024}$. The remaining combinations will give at least one head; thus probability of getting at least one head is $1 - \left(\frac{1}{1024}\right) = \frac{1023}{1024} \approx 0.999$.

Homework

1) You take the standard card deck and draw one card. What is the probability that the card will be
    (a) Queen of hearts
    (b) Either a queen or a hearts card
    (c) A red card
    (d) A picture card (a jack, queen, king, ace)
    (e) A picture card other than the queen of hearts

2)   (a) What is the probability that when we toss a coin 4 times, there will be no heads?
      (b) A and B are playing the following game. They toss a coin 4 times; if there are no heads, A wins and B pays him $10. Otherwise A loses and he pays $1 to B. Would you prefer to play for A or for B in this game?

3)   (a) What is the probability that when we roll two dice, at least one will be a 6?
      (b) A and B are playing the following game. They roll two dice; if at least one is a 6, A wins and B pays her $5. Otherwise A loses and she pays $1 to B. Would you prefer to play for A or for B in this game?

4)   (a) What is the probability that if we roll 3 dice all the numbers will be odd?
      (b) A and B are playing the following game. They roll 3 dice; if all numbers are odd, A wins, and B pays him $5. Otherwise A loses and he pays $1 to B. Would you prefer to play for A or for B in this game?

5) Suppose there is an equal chance that a boy or a girl is born; what is the probability that the first five babies born next Saturday morning at St. Charles Hospital will be girls? That at least one of the five babies will be a girl?

6) In a certain club of 30 people, they are selecting a president, a vice-president, and a treasurer (they all must be different people: no one is allowed to take two posts at once). How many ways are there to do this?

7) In a group of 100 students, 28 speak Spanish, 30 speak German, 42 speak French; 8 students speak Spanish and German, 10 speak Spanish and French, 5 speak German and French and 3 students speak all 3 languages. How many students do not speak any one of the three languages? [Note: when it says that 28 students speak Spanish, this includes the 8 who speak Spanish and German; similarly for all other combinations.]

8) How many whole numbers between 1 and 1000 are divisible by 3? by 5? by 15? are not divisible by either 3 or 5?