

Math 6a/b: Homework 4

Homework #4 is due October 21. *Please, write clearly which problem you are solving and show all steps of your solution.* Some of the problems in this assignment are from the island of Knights and Knaves. Remember, Knights always tell the truth, and Knaves always lie. You can find these problems and many more at: <http://philosophy.hku.hk/think/logic/knights.php>

1. Solve the following equations:

(a) $2x - 22 = 3(1 - x)$ (b) $1 - \frac{2}{7}x = \frac{1}{7}x$ (c) $1 - 8(1 - x) = 7x - 8$

2. On the island of Knights and Knaves, you meet three inhabitants: Bob, Mel, and Peggy. Bob says that it's not true that Peggy is a knave." Mel says that "Peggy is a knight or Bob is a knave." Peggy claims "both I am a knight and Bob is a knave."

3. Many trucks carry the message: "If you do not see my mirrors, then I do not see you." Can you rewrite this in an equivalent form, without using the word 'not'?

4. Define a new logical operation *XOR* (exclusive *OR*) as follows: $A \text{ XOR } B$ is *only* true if exactly A is true or B is true, but not when both are true.

(a) Write the truth table for $A \text{ XOR } B$

(b) Can you express *XOR* using only *AND*, *OR*, and *NOT* (that is, write a formula equivalent to $A \text{ XOR } B$ using only *AND*, *OR*, and *NOT*)?

5. (a) Write truth tables for formulas $A \text{ AND } (B \text{ OR } C)$ and $(A \text{ AND } B) \text{ OR } C$ (hint: there will be 8 rows in the table). Are these formulas equivalent? (i.e. do they always give the same answer?)

(b) The waiter in a restaurant tells you: "our fixed price dinner includes soup and appetizer or salad." Denoting

A = your dinner will include soup

B = your dinner will include appetizer

C = your dinner will include salad

What would be the correct way to write his statement using letters A , B , C and logical operators *AND*, *OR*?

6. On the island next to the island of knights and knaves there are three kinds of people:

Knights, who always tell the truth,

Knaves, who always lie

Normal people, who sometimes lie and sometimes tell the truth

On that island, you meet 3 people, A , B , and C , one of whom is a knight, one a knave, and one normal (but not necessarily in that order). They make the following statements:

A : "I am normal"

B : "That is true"

C : "I am not normal"

What are A , B , and C ?