SchoolNova, Math 5b
Homework 10
Binary and Base-4 Representation of Numbers
January 13, 2018

Please provide sufficient details about how you solved the problem. More difficult problems are marked with a $*$. If unable to solve a problem, please present your thoughts and any partial solution.

A Binary Numbers

In binary or *base-2* number system, we only use 2 digits, namely 0 and 1. So for example, the binary number 11001 equals $1 \times 2^4 + 1 \times 2^3 + 0 \times 2^2 + 0 \times 2^1 + 1 \times 2^0 = 25$. We also wite this as $11001_2 = 25_{10}$. The computers use binary numbers for internal representation and processing. Each digit in a binary number is referred to as a *bit*. Here are some decimal numbers and their equivalent binary numbers:

Decimal	**Binary**
0	000
1	001
2	010
3	011
4	100
5	101
6	110
7	111

A.1 Addition of Binary Numbers

The addition of binary numbers is done very similar to the decimal numbers, where one-digit numbers are added as follows:
$0 + 0 = 0$
$0 + 1 = 1$
$1 + 0 = 1$
$1 + 1 = 0$, carry 1.
So for example, we add the numbers 25 and 3 as follows:

$$
\begin{array}{ccccccc}
 & & & 1 & 1 & & \\
 & 1 & 1 & 0 & 0 & 1 & \quad 25 \\
+ & & & & 1 & 1 & \quad 3 \\
\hline
 & 1 & 1 & 1 & 0 & 0 & \quad 28
\end{array}
$$

A.2 Subtraction of Binary Numbers

One way of representing negative numbers in binary is using the *two's- complement* representation. This makes subtraction easy, as now you can add the negative number to the positive number, using the rules of addition. In this notation, one bit (the leftmost bit) indicates the sign of the number. So for example, 3-bits which would represent the numbers 0 through 7 for unsigned numbers, would now represent the numbers -4 through 3 in two's-complement notation. The table below shows these numbers, utilizing 3-bits.
In two's-complement, positive numbers have the same bit representation. To obtain the binary form of a negative number, for example, -3, reverse the bits of $+3$ and add 1.

Decimal	**Binary**
3	011
2	010
1	001
0	000
-1	111
-2	110
-3	101
-4	100

In this example, we perform the operation $3 - 1$ with $3 + (-1)$. The leftmost bit after the 3-bits is discarded.

$$
\begin{array}{ccccc}
 & 1 & 1 & & \\
 & 0 & 1 & 1 & \quad 3 \\
+ & 1 & 1 & 1 & \quad \text{-1} \\
\hline
 & 0 & 1 & 0 & \quad +2
\end{array}
$$

B Assignment - Part I

1. Find the value of x if

   (a) $2^x + 2^x + 2^x = 192$

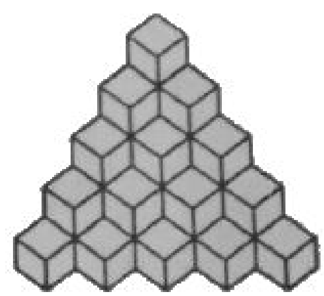
   (b) $2^3 + 2^x = 2^4$

   (c) $8^{255} = 32^x$

2. Using results from algebra, find the prime factorization of the following number: $99^2 - 9^2$.

3. $*$ The notation $a|b$ means a divides b, with a remainder 0. Show that if $a|b$ and $b|c$, then $a|c$.

4. The tower in the picture is made up of five horizontal layers of cube with no gaps. How many individual cubes are in the tower?



C Assignment - Part II

1. Write all numbers from 0 through 30 in base-2 and base-4.

2. Convert each of the following binary numbers to its equivalent decimal number:

   (a) 1001 (b) 11111 (c) 101010 (d) 11110000 (e) 100001

3. Find the base-2 representation of each of the following decimal numbers:

   (a) 17 (b) 64 (c) 75 (d) 222 (e) 1049

4. Perform the following binary additions:

   (a) 1+1001 (b) 1001+1001 (c) 101010+10101 (d) 101010+10101

5. Yashas runs each day for 45 minutes. Write this number in base-2, and in base-4.