SchoolNova, Math 5b
Homework 1
Prime Factorization, GCD, LCM, Factorials
September 16, 2018

Please provide sufficient details about how you solved the problem. More difficult problems are marked with a *. If unable to solve a problem, please present your thoughts and partial solution.

1. Find the prime factorization of the following numbers: (a) 1245 (b) 1352 (c) 1683

2. Find the Least Common Multiple (LCM) and Greatest Common Divisor (GCD) of the following numbers, using prime factorization:

   - 42 and 52.
   - 51 and 340.
   - 1012 and 1232.

3. Consider the number $2 \times 2 \times 2 \times 5 \times 5 \times 7 \times 11$. In how many zeros does it end? (Try doing it without performing the multiplication).

4. Said Anne to Betty: "If you give me one marble, we will each have the same number of marbles."
   Said Betty to Anne: "If you give me one marble, I will have twice as many marbles as you will have."
   How many marbles did Anne have (before the exchange)?

5. A merchant came to the market to sell some eggs. A first buyer took half her eggs plus another 1/2 egg. A second buyer took half the remaining eggs plus another 1/2 egg. A third buyer took only what was left over: 1 egg. A buyer never takes a broken egg home. How many eggs were there initially?

6.* Without multiplying all the terms, show that

   (a) $10! = 6!\,7!$
   (b) $10! = 7!\,5!\,3!$
   (c) $16! = 14!\,5!\,2!$

7.* Let a be a counting number.

   (a) What is the GCD of a and $a+1$?
   (b) What is the GCD of a and $a+2$?

8.* It is known that $a+1$ is divisible by 3. Show that $4+7a$ is also divisible by 3.