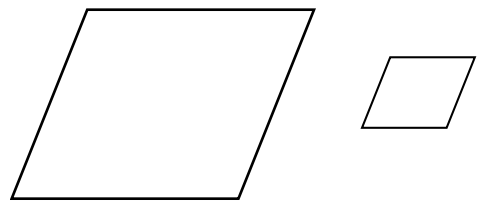
Math 5: Handout 24
Geometry 5.

Similar figures

Two figures are called *similar* of one of them can be obtained from the other by rescaling, or stretching (equally in all directions).



In similar figures all angles are the same, and all lenghts are increased in the same proportion. For example, if triangles ABC and $A'B'C'$ are similar, then

$$\angle A = \angle A', \quad \angle B = \angle B' \quad \angle C = \angle C'$$
$$A'B' = kAB, \quad A'C' = kAC, \quad B'C' = kBC$$

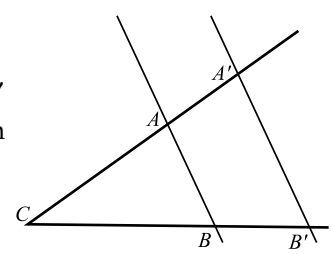
The number k is called the similarity coefficient.

Similarity tests for triangles

Axiom 1 (AAA Rule)**.** *If three angles of one triangle are equal to corresponding angles of another triangle, then the triangles are similar.*

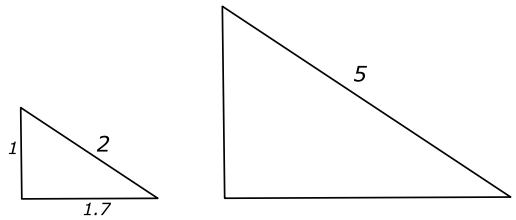
For example, if ABC, $A'B'C'$ is a right triangles with $\angle C = \angle C' = 90°$, $\angle A = \angle A' = 30°$, then these triangles are similar.

Another example is shown in the figure to the right. Let lines $l = AB, l' = A'B'$ be parallel. Then triangles ABC and $A'B'C$ are similar: indeed, by the theorem about alternate angles, we have $\angle A = \angle A'$, $\angle B = \angle B'$.

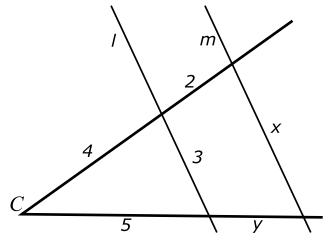


Homework

1. Find the missing lengths in the second triangle (triangles are similar).



2. Find the lengths x, y in the figure (lines l, m are parallel).



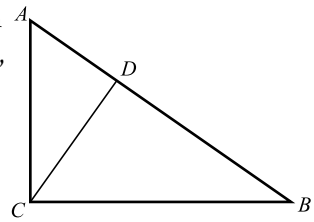
3. In the story *The Musgrave Ritual*, Sherlock Holmes was following directions of an ancient manuscript that required him to measure that many steps from the tip of a shadow of an ancient elm at specific time of day. Unfortunately, by the time of the story the elm was long gone; however, Holmes was told that tis height had been 64 feet. Here is how he proceeded:

   "…there was no real difficulty. I went with Musgrave to his study and whittled myself this peg, to which I tied this long string with a knot at each yard. Then I took two lengths of a fishing-rod, which came to just six feet, and I went back with my client to where the elm had been. The sun was just grazing the top of the oak. I fastened the rod on end, marked out the direction of the shadow, and measured it. It was nine feet in length.

   "Of course the calculation now was a simple one. If a rod of six feet threw a shadow of nine, a tree of sixty-four feet would throw one of …"

   Can you complete the sentence and find how long the shadow of the elm would be?

4. The figure shows right triangle ABC: $\angle C = 90°$, with $AC = 4$ cm, $AB = 5$ cm. The line CD is the altitude, i.e., it is perpendicular to side AB.



   a. Show that triangles ABC and ACD are similar.
   b. Find lengths BC, CD.

5. Given a rectangle with sides 2 cm and 4 cm, can you:
   a. Cut it into three pieces that can be rearranged to get a right triangle?
   b. Cut it into three pieces that can be rearranged to get a square?
6. In a rhombus, two diagonals are 16 cm and 12 cm. Find the area and perimeter of the rhombus.
7. Solve the equation $7x - 11 = 5x - 3$.