Math 5: Handout 23
Geometry 4.

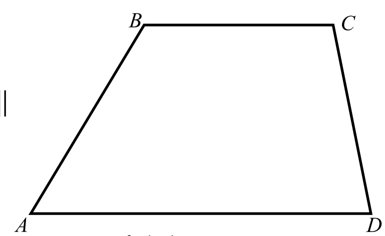
Special quadrilaterals: parallelogram, rhombus, trapezoid

Recall that a **parallelogram** is a quadrilateral in which opposite sides are parallel. Here are some of the properties of parallelograms; all of them were either proved before or will be proved in this homework.

1. In a parallelogram, opposite sides are equal. Conversely, if $ABCD$ is a quadrilateral in which opposite sides are equal: $AB = CD$, $BC = AD$, then $ABCD$ is a parallelogram.
2. In a parallelogram, diagonals bisect each other, i.e. the intersection point of two diagonals is the midpoint of each of them. Conversely, if $ABCD$ is a quadrilateral in which diagonals bisect each other, then $ABCD$ is a parallelogram.

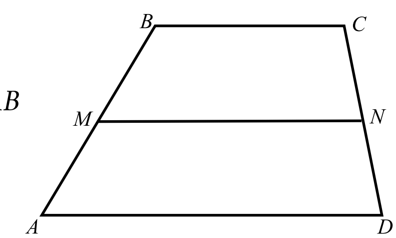
A **rhombus** is a quadrilateral in which all four sides are equal. By property 1 above, any rhombus is also a parallelogram, so all the above properties hold. In addition, in a rhombus diagonals are perpendicular (shown in previous homework).

A **trapezoid** is a quadrilateral in which one pair of opposite sides are parallel: $AD \parallel BC$. These parallel sides are usually called **bases**.

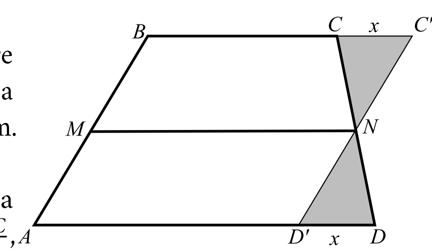


A trapezoid does not have as many useful properties as a parallelogram, but here is one useful thing.

Theorem. *Let $ABCD$ be a trapezoid with bases AD, BC. Let M be midpoint of side AB and N — midpoint of side CD. Then $MN \parallel AD$ and $MN = \frac{AD+BC}{2}$.*



Proof. In homework exercise 2, you will show that it works for a parallelogram. Let us use it.
Draw a line $C'D'$ through point N parallel to AB. Then the two shaded triangles are congruent by ASA, so N is also the midpoint of $C'D'$. On the other hand, $ABC'D'$ is a parallelogram, so MN is the line connecting midpoints of two sides of a parallelogram. Thus, by exercise 2, $MN \parallel AD$, $MN = AD' = BC'$.
Denote $x = CC' = BB'$. Then $BC' = BC + x$, $AD' = AD - X$. Since $ABC'D'$ is a parallelogram, $BC' = AD'$, so $BC + x = AD - x$. Solving for x, we get $x = \frac{AD-BC}{2}$, so $MN = BC' + BC + x = \frac{AD+BC}{2}$. □



Area

Recall that the area of a rectangle is (length)×width, and area of a right triangle with legs a, b is $\frac{1}{2}ab$ (because putting together two such triangles we get a rectangle with sides a, b). Here are more formulas:

Area of a triangle with base b and height h is $\frac{1}{2}bh$
Area of a parallelogram with base b and height h is bh
Area of a trapezoid with bases a, b and height h is $h \times \frac{a+b}{2}$

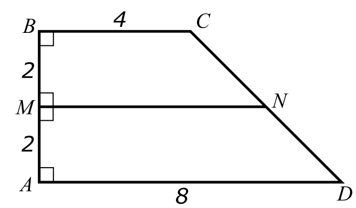
Homework

Warning: in this homework, more than ever, you will need results of previous exercises when doing the next one. So when doing, say, exercise 2, see if you can make use of exercise 1.

1. Let $ABCD$ be a quadrilateral such tath $AB = CD$, $AB \parallel CD$. Show that then $ABCD$ is a parallelogram. [Hint: show that tirangles $\triangle ABD$, $\triangle CDB$ are congruent.]
2. Let $ABCD$ be a parallelogram, and let M, N be midpoints of sides AB, CD. Show that then $AMND$ is a parallelogram, and deduce from this that $MN \parallel AD$, $MN = AD$.
3. a. Show that if in a quadrilateral $ABCD$ diagonals bisect each other (i.e., intersection point is hte midpoint of each of the diagonals), then $ABCD$ is a parallelogram. [Hint: find some congruent triangles in the figure.]
   b. Show that if in a quadrilateral $ABCD$ diagonals bisect each other and are perpendicular, then it is a rhombus.
4. To check whether a piece of paper is a square, John folds it along a diagonal. If the corners match, he decides it is a square. Is he right? What if he folds along both diagonals?
5. Can you cut a trapezoid into pieces from which you can construct a rectangle?

*6. Let ABD be a triangle, and M, N —midpoints of sides AB, BD. Show that then $MN \parallel AD$, $MN = \frac{1}{2}AD$. [Hint: think of the triangle as a trapezoid in which the top base is so small it becomes a single point. Try to see if the proof given above for trapezoids will work for a triangle, too.]

7. Find all lenghts, angles, and area in the figure shown to the right.



8. Suppose you have a large supply of tiles, all of the same size and shape — namely, a parallelogram. Can you tile a plane with these tiles? Can you find different ways of doing this? What if instead of a parallelograms you have trapezoids?