Math 5: Handout 12
Powers.

Powers

Notation (n is a whole number):

$$a^n = \underbrace{a \cdot a \cdot \dots \cdot a}_{n \text{ times}}$$

In particular,

$$\begin{aligned} a^1 &= a \\ a^2 &= a \cdot a \qquad \text{reads "} a \text{ square"} \\ a^3 &= a \cdot a \cdot a \qquad \text{reads "} a \text{ cube"} \end{aligned}$$

Properties:

$$\begin{aligned}
(ab)^n &= \underbrace{ab \cdot ab \dots ab}_{n \text{ times}} = \underbrace{a \cdot a \dots a}_{n \text{ times}} \cdot \underbrace{b \cdot b \dots b}_{n \text{ times}} = a^n b^n \\
a^m a^n &= \underbrace{a \cdot a \cdot \dots \cdot a}_{m \text{ times}} \cdot \underbrace{a \cdot a \cdot \dots \cdot a}_{n \text{ times}} = \underbrace{a \cdot a \cdot \dots \cdot a}_{m+n \text{ times}} = a^{m+n} \\
\left(\frac{a}{b}\right)^n &= \underbrace{\frac{a}{b} \cdot \frac{a}{b} \cdot \dots \cdot \frac{a}{b}}_{n \text{ times}} = \frac{a^n}{b^n} \qquad (1) \\
\frac{a^m}{a^n} &= \frac{\overbrace{a \cdot a \cdot \dots \cdot a}^{m \text{ times}}}{\underbrace{a \cdot a \cdot \dots \cdot a}_{n \text{ times}}} = \underbrace{a \cdot a \cdot \dots \cdot a}_{m-n \text{ times}} = a^{m-n} \qquad \text{if } m > n
\end{aligned}$$

These rules are especially useful for multiplication of large numbers, written in “scientific notation”, like $5.12 \times 10^6 = 5.12 \times 1000000 = 5120000$. For example,

$$1.2 \times 10^4 \times 3 \times 10^8 = 1.2 \times 3 \times 10^4 \times 10^8 = 3.6 \times 10^{12}$$

Homework problems on back

Homework

For the homework, let me remind you that you should write solutions, showing your calculations and your reasoning — not just answers! Do not write on this homework assignment — use a separate sheet of paper instead; leave the homework assignment in your folder for future reference.

1. Solve the following equations

$$\text{(a) } 5 - x = -4 - 2x$$
$$\text{(b) } 7 - 2(1 - x) = -5$$
$$\text{(c) } \frac{x-2}{x-1} = 3$$

2. Father is 32 years old and son is 5 years old. In how many years will the father be 10 times older than the son?
3. Simplify the following expression:
$$\frac{(x^2y^2)\cdot x^3}{x^2y^5}$$
4. Let $a = 2 \cdot 10^8$, $b = 10^5$. Compute $a^2 \cdot b$, $\frac{a}{b}$, $a^2 \div b^3$.
5. How many cubic centimeters are there in one cubic kilometer? (1 km =1000 m, 1 m=100 cm).
6. It is known that $2^{10} = 1024$, which is very close to 10^3. Using this, can you estimate what is the value of 2^{20}? 2^{32}?

*7. Consider the sequence $7, 7^2, 7^3, \dots 7^n \dots$

   a. Show that there will be two numbers in this sequence which have the same last two digits. [Hint: pigeonhole principle!]
   b. Show that from some moment, the last two digits of numbers in this sequence will start repeating periodically.

8. (from *101 puzzles in thought and logic*, by C. R. Wylie)
   Clark, Jones, Morgan, and Smith are four men whose occupation are butcher, druggist, grocer, and policeman, though not necessarily in that order.
   Clark and Jones are neighbors and take turns driving each other to work
   Jones makes more money than Morgan
   Clark beats Smith regularly at bowling
   The butcher always walks to work
   The policeman doesn't live near the druggist
   The only time the grocer and the policeman ever meet is when the policeman arrested the grocer for speeding
   The policeman makes more money than the druggist or the grocer
   What is each man's occupation?