

Math 5: Handout 11

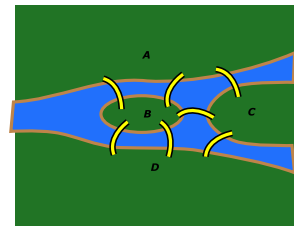
Bridges of Königsberg.

Today is the last class of this year. We will meet again in January; the first class will be on Jan. 6.

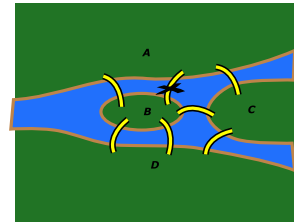
During the class we talked about the bridges of Königsberg. We found that the key to answering this kind of problems lies in counting the number of islands with odd number of bridges — or, if you replace each island by a point and each bridge by an arc, the number of points which have an odd number of arcs coming into them. If there are at most two such points, the problem can be solved; if more than two, it can not be solved.

Classwork

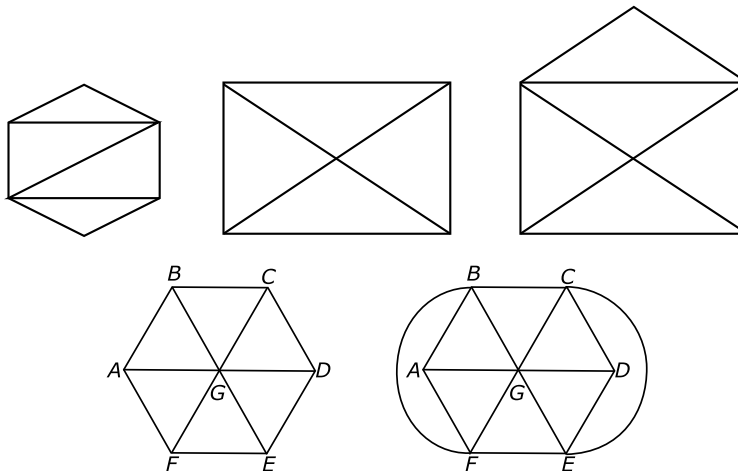
1. The figure to the right shows a map of a city with islands and bridges. Is it possible to complete a walk in this city so that you walk on each of the seven bridges exactly once? (You may start anywhere you like, and you do not have to come back to the starting point.)



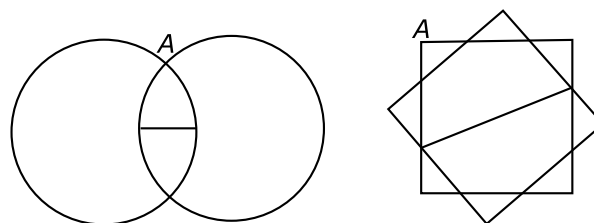
2. Once, flood destroyed one of the bridges between river bank A and island B; the new map (with the destroyed bridge crossed out) is shown here. Is it now possible to complete a walk in this city so that you walk on each of the seven bridges exactly once?



3. Which of the figures below can be drawn without lifting your pencil from the paper and drawing every line exactly once, without retracing?



4. Which of the figures below can be drawn without lifting your pencil from the paper **starting at point A** and drawing every line exactly once, without retracing? What if you start at some other point?



Homework

The assignment below is optional. It is not so much a homework as some nice problems which you might enjoy doing over the winter break. I tried to make it more fun than usual...Happy holidays!

1. Suppose you need to find a 100-digit number; as before, you can ask “yes/no” questions. However, the person answering can lie once.

Below you can find two strategies for finding this number.

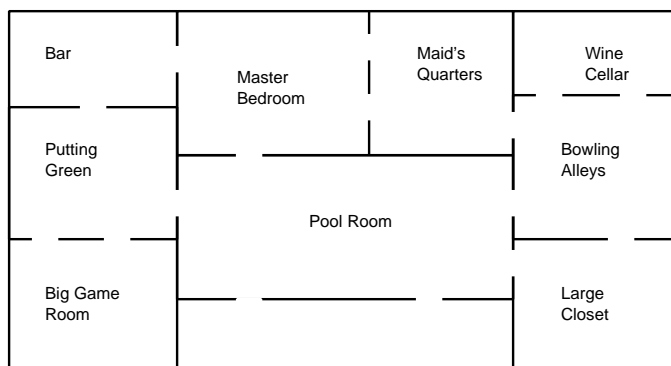
- a. Let us first find the first digit, by asking “is the first digit less than 5?”, etc; once we think we got it, we will double-check it by asking “is the first digit ...” once again. If the answers match, we move on to the second digit; if not, we repeat the questions about the first digit.
- b. Similar to the first one, but instead of finding one digit at a time, we find two digits at a time, by asking “is the number formed by the first two digits less than 50?”, etc; as before, once we think we got it, we will double-check it by asking “is the number formed by the first two digits equal to ...” once again. If the answers match, we move on to the next pair of digits; if not, we repeat the questions about the first pair.

Which of the strategies is faster? Can you think up an even faster strategy?

2. There are 6 coins one of which is fake (its weight is different from the weight of real one, but neither of the weights are known). You are given scales which show the total weight of the coins being weighed. What is the fastest way to find the fake coin?
3. Let us write all numbers from 1 to 1000 on a piece of paper.
 - a. How many digits 1 will there be on this piece of paper? How many twos?
 - b. What is the sum of digits of all numbers from 1 to 1000?
4. Brown, Clark, Jones, and Smith are the names of the men who hold, though not necessarily respectively, the positions of accountant, cashier, manager, and president in the First National Bank of Fairport.
 - Although the cashier beats him consistently, the president will play chess with no one else in the bank.
 - Both the manager and the cashier are better chess players than the accountant.
 - Jones and Smith are next door neighbors and frequently play chess together in the evening.
 - Clark plays a better game of chess than Jones.
 - The accountant lives near the president but not near any of the others.

What position does each man hold?

5. The figure below shows a plan of a house.



Is it possible to enter this house, walk through it going through every door exactly once, and then exit again through the other door? If not, could it be done if you are allowed to miss one door, not going through it? [Hint: it may be easier if instead of this picture, you make a new one, replacing each room (and also the outside of the house) by a point, and drawing a line connecting two points for every door.]