Math 5: Handout 10
Binary numbers. n-ary numbers.

Today we talked more about binary numbers. We discussed arithmetic operations with binary numbers. We also discussed that in computers, letters and other symbols are written as sequences of 0 and 1 (bits); since there 2^n such sequences of length n, and there are 26 letters in English alphabet, we need at least 5 bits ($2^5 = 32$) for each English letter. If we want to have lower- and upper-case letters, punctuation, numbers, accented letters such as é, we need more; in real life, people use 8 bits per symbol (called "byte").

The correspondence between actual letters and their codes, i.e. sequences of 0 and 1, is called *encoding*. The most common encoding (Latin 1, aka ISO 8859-1) is shown in the attached table. In this table, rows correspond to the last 4 bits, and columns, to the first four bits. For example, lower case letter "a" has code 01100001.

We also touched on other bases. For example, in base 3, we only use digits 0, 1, 2, and they correspond to powers of 3:

$$21021_3 = 2 \cdot 81 + 1 \cdot 27 + 0 \cdot 9 + 2 \cdot 3 + 1 \cdot 1 = 250_{10}$$

If the base is larger than 10, then in addition to digits $0 \ldots 9$ we use letters A, B, etc. For example, in base 16, we use digits $1, \ldots, 9$ and letters $A = 10, B = 11, C = 12, D = 13, E = 14, F = 15$. The digits correspond to powers of 16:

$$D4B_{16} = D \cdot 256 + 4 \cdot 16 + B \cdot 1 = 13 \cdot 256 + 4 \cdot 16 + 11 \cdot 1 = 3403_{10}$$

Homework

1. You have scales, a 1 gram weight and a large bag of sugar. What would be the fastest way to measure exactly 8 grams of sugar? exactly 128 grams? exactly 100 grams?
2. What is the largest number that can be written as a 5-digit binary number? (Hint: what is the smallest 6-digit binary number?)
3. Is it possible to encode every letter of English alphabet by a 4-digit binary number? You can choose any way you like — for example encoding A as 0000, B as 0001, or by any other method.

   Would it be possible if we used 5-digit binary numbers?
4. A car has traveled 125 miles during some period. During the same period, another car, which is faster by 10 mph, has traveled 150 miles. What is the speed of the faster car?
5. Do the following arithmetic operations with binary numbers. Try doing them without converting the numbers to decimal form.

   a. $110101_b + 111011_b$ b. $10101_b \times 1011_b$ c. $(10101_b + 1101_b) \times 10110_b$
6. **[Skip for now!]** The following is a beginning of a computer file. Can you decode it (assuming it is written in the standard, Latin 1, encoding)?

   01010100 01101111 01110000 00100000 01110011 01100101 01100011 01110010 01100101 01110100 00001010
7. **[Skip for now!]** In order to allow computers to deal with different languages, computer scientists have developed so-called Unicode, a standard list of symbols covering virtually all human languages, from Armenian to Vietnamese. In particular, it includes Latin letters, Cyrillic letters, Chinese characters (hanzi), Emoji, and more.

   The latest revision of Unicode contains about 96,000 symbols. If we want to represent each of them by a sequence of 0 and 1, would it be enough to use 16 bits (0s and 1s) for each symbol? If not, what is the smallest number of bits per symbol one would need?

8. Perform calculation in the base 4:

   a. 333×2 b. $1111 - 222$ c. $3231 - 1321$

9. **[Skip for now!]**Perform calculation in the base 13:

   a. $9999 + 2222$ b. $1A2B3C + 999999$ c. $9C138 - 5A001$

10. Fish head weighs as much as the tail and half of the body together. The body weighs as much head and tail together, and the tail weighs 1 kg. How heavy is the fish?

11. You are given several coins, one of which is fake. The weight of the fake one is different from the weight of the real ones, but it is not known whether it is heavier or lighter. Can you find whether the fake one is heavier or lighter than the real one using two measurements with the scales (2 platforms, no weights) if the total number of coins is:

    a. 100 b. 99 c. 98

    You do not have to find which coin is the fake one.

6.2 Code table

For each character in the set the code table (table 2) shows a graphic symbol at the position in the code table corresponding to the bit combination specified in table 1.

The shaded positions in the code table correspond to bit combinations that do not represent graphic characters. Their use is outside the scope of ISO/IEC 8859; it is specified in other International Standards, for example ISO/IEC 6429.

Table 2 – Code table of Latin alphabet No. 1

				b_8	0	0	0	0	0	0	0	0	1	1	1	1	1	1	1	1
				b_7	0	0	0	0	1	1	1	1	0	0	0	0	1	1	1	1
				b_6	0	0	1	1	0	0	1	1	0	0	1	1	0	0	1	1
				b_5	0	1	0	1	0	1	0	1	0	1	0	1	0	1	0	1
b_4	b_3	b_2	b_1		00	01	02	03	04	05	06	07	08	09	10	11	12	13	14	15
0	0	0	0	00			SP	0	@	P	`	p			NBSP	°	À	Ð	à	ð	0
0	0	0	1	01			!	1	A	Q	a	q			¡	±	Á	Ñ	á	ñ	1
0	0	1	0	02			"	2	B	R	b	r			¢	²	Â	Ò	â	ò	2
0	0	1	1	03			#	3	C	S	c	s			£	³	Ã	Ó	ã	ó	3
0	1	0	0	04			$	4	D	T	d	t			¤	´	Ä	Ô	ä	ô	4
0	1	0	1	05			%	5	E	U	e	u			¥	µ	Å	Õ	å	õ	5
0	1	1	0	06			&	6	F	V	f	v			¦	¶	Æ	Ö	æ	ö	6
0	1	1	1	07			'	7	G	W	g	w			§	·	Ç	×	ç	÷	7
1	0	0	0	08			(	8	H	X	h	x			¨	¸	È	Ø	è	ø	8
1	0	0	1	09			)	9	I	Y	i	y			©	¹	É	Ù	é	ù	9
1	0	1	0	10			*	:	J	Z	j	z			ª	º	Ê	Ú	ê	ú	A
1	0	1	1	11			+	;	K	[	k	{			«	»	Ë	Û	ë	û	B
1	1	0	0	12			,	<	L	\	l	|			¬	¼	Ì	Ü	ì	ü	C
1	1	0	1	13			-	=	M	]	m	}			SHY	½	Í	Ý	í	ý	D
1	1	1	0	14			.	>	N	^	n	~			®	¾	Î	Þ	î	þ	E
1	1	1	1	15			/	?	O	_	o				¯	¿	Ï	ß	ï	ÿ	F
					0	1	2	3	4	5	6	7	8	9	A	B	C	D	E	F	hex