Math 5: Handout 7
Negative Numbers. Absolute Value.

Negative numbers: a review

Numbers to the left of zero on the number line are called negative. They are less than 0, and we write the "–" in front of them. The numbers to the right from zero are positive.

Addition and subtraction.

If we add a positive number to any number, we move to the right along the number line. If we add a negative number to any number, we move to the the left along the number line. So, adding (-5) is moving 5 units to the left on the number line — which is the same as subtracting 5. This rule holds in general:

$$a + (-b) = a - b$$

Addition is commutative: $a+b = b+a$, whether numbers a, b are positive or negative. So $(-3)+7 = 7+(-3) = 7-3 = 4$.

Opposites.

Pairs of numbers -1 and 1, -2 and 2, -3 and 3 etc. are called the opposites. They lie at the same distance from zero on the number line, but in the opposite directions. For any number x (whether positive or negative), we will denote by $-x$ the opposite of x. For example, $-(-2)$ is the opposite of negative 2, which is equal to 2.

Absolute value.

The distance of a number from zero on the number line is called the absolute value of a number. The symbol for absolute value is $|\ |$. For example, $|4| = |-4| = 4$. It holds in general: for any positive x, we have $|x| = |-x| = x$.

Equations with absolute values

An equation like $|x| = 5$ has two solutions: $x = 5$ and $x = -5$. An equation like $|x - 1| = 4$ also has two solutions: $x - 1 = 4$ (which gives $x = 5$) or $x - 1 = -4$, which gives $x = -4 + 1 = -3$.

Experiment

Find any round or cylindrical object. Measure a diameter of it and a circumference. Divide the circumference by the diameter — the value you get should be close to π.

Please bring the picture of your object of choice to class (either print it, or just have it on a mobile device, if you have one!), as well as your measurements and what your final result is.

Homework

1. Compute

$$\begin{array}{lll} (-4)+(-7)= & 3+(-6)+(-7)= & (-2)+5-4)= \\ -3+(-1)+5= & 14+(-7)+(-3)= & |(-5)+4|= \\ |(-3)+(-7)|= & -|3+(-6)|= & -|(-2)+(-6)|+1= \end{array}$$

2. Solve the equations:

$$\begin{array}{ll} \text{a) } x-12=-17 & \text{b) } -13-y=-9 \\ \text{c) } 5+2z=-19 & \text{d) } z+|-6|=-15 \\ \text{e) } |x|=3 & \text{f) } |x-8|=12 \end{array}$$

3. Two cities, A and B, are on the same river. It takes the motorboat 2.5 hours to travel from A to B, and 5 hours to travel from B to A (because B is downstream from A, so going back it has to go against the flow). How fast is the river flow if the speed of the boat (in still water, say on a lake) is 12 km/h? How far away are the cities? [Hint: if the flow is x km/h, then going down stream the boat makes $12+x$ km/h....)

4. A swimming pool can be filled by an inlet pipe in 12 hours and emptied by an outlet pipe in 15 hours.

   One day the pool is empty and the owner opens the inlet pipe to fill it. However, he forgets to close the outlet pipe. How long will it take the pool to fill?

5. Here are phrases in Swahili with their English translations:

- atakupenda – He will love you.
- nitawapiga – I will beat them.
- atatupenda– He will love us.
- anakupiga – He beats you.
- nitampenda – I will love him.
- unawasumbua – You annoy them.

Translate the following into Swahili:

- You will love them.
- I annoy him.

6. A bank clerk has 60 coins which should be identical but one of them is fake. The fake one looks the same as all other coins but is lighter. Using the balance scales (but no weights — so you have to put coins on both platforms), what is the fastest way of finding the fake coin? What if you do not know whether the fake coin is lighter or heavier than the real one?