

Math 5: Handout 6

Fractions and Decimals. More Word Problems

Today's material

Sets

Sets are collections of similar elements. In mathematics, we usually talk about sets of numbers. The main sets are:

- \mathbb{N} : Set of natural numbers, i.e. $1, 2, 3, \dots$;
Operations: $+$, \times .
- \mathbb{Z} : Set of integer numbers, i.e. $\dots, -3, -2, -1, 0, 1, 2, 3, \dots$;
Operations: $+$, $-$, \times .
- \mathbb{Q} : Set of rational numbers, i.e. numbers that can be written as a fraction
Operations: $+$, $-$, \times , \div .

Fractions and Decimals

We also talked about how to convert fractions to decimals (see the long division on the right, where we attempt to convert $\frac{2}{7}$ into a decimal. We keep doing long division until the remainder repeats. Every fraction can be converted to a decimal which will either be finite, or it will be infinite, but there will be a sequence of numbers which constantly repeats itself.

$$\frac{2}{7} = 0.285714285714 \dots = \overline{0.285714}.$$

We also introduced the *reciprocal*. For a number a , a reciprocal $r(a)$ is such a number that $a \times r(a) = 1$. For example, $r(\frac{3}{8}) = \frac{8}{3}$, because $\frac{3}{8} \times \frac{8}{3} = 1$.

Classwork

1. The books on mathematics are 4 times more expensive than the books on English. Mary can buy 3 books on mathematics and have 4 dollars left, or 10 books on English and have 10 dollars left. What is the price of the book on Mathematics? And on English?
2. The watermelon is three times as expensive as the honeydew. John can buy 2 watermelons and have 7 dollars left or 4 honeydews and have 13 dollars left. How much does the honeydew cost? And how much is the watermelon?
3. A pen is three times as expensive as the notebook. If Bill buys 5 pens, he will have 6 dollars. If Bill buys 3 pens and 3 notebooks, he will have 12 dollars left. How much extra money will he need, if he decides to buy 7 pens and 4 notebooks?
4. A hot water tap fills the bath in 5 minutes. The cold water tap fills the bath in 3 minutes. With both taps open, how long will it take to fill the bath?

$$\begin{array}{r} 0.2857142 \\ 7 \overline{) 2.0000000} \\ \underline{1.4} \\ 60 \\ \underline{56} \\ 40 \\ \underline{35} \\ 50 \\ \underline{49} \\ 10 \\ \underline{7} \\ 30 \\ \underline{28} \\ 20 \end{array}$$

Homework problems on back

Homework

1. Open parentheses and collect the like terms:
 - a. $3a(a - 3b) - (2a^2 - 6ab) + 2b(-a - b)$
 - b. $4a(ba + ac) - b(2a^2 - 2a) + 7ab - 11c^2a - 19aca + 15cac$
2. Find the reciprocals of the following numbers.
 - a. $r(\frac{13}{7})$
 - b. $r(3)$
 - c. $r(1\frac{6}{7})$
 - d. $r(3\frac{1}{5})$
 - e. $r(0.1)$
 - f. $r(1.1)$
3. Convert the following fractions into decimals:
 - a. $\frac{1}{8}$
 - b. $\frac{5}{6}$
 - c. $\frac{1}{7}$
 - d. $\frac{1}{13}$
4. One bookshelf has 70 books and the other one 36. How many books should you move from the first shelf to the second in order to have equal number of books on both?
5. Can you solve the previous problem if the first bookshelf has 70 books, and the second one has 35 books?
6. The car cost \$12,000 more than the motorbike. At the same time, the car is 4 times as expensive as the motorbike. How much do the car and the motorbike cost?
7. If Alice comes to the store with 27 dollars and buys 4 jumping ropes, she will have the same amount of money left as if she comes to the store with 42 dollars and buys 7 jumping ropes. What is the price of the jumping rope?
8. Susan comes to the store to buy oil and vinegar for the salad dressing for a huge party. A bottle of oil is four times as expensive as a bottle of vinegar. If Susan decides to buy 2 bottles of oil and 3 bottles of vinegar, she will have 8 dollars left. If she decides to buy 4 bottles of oil and 2 bottles of vinegar, she will need 6 extra dollars. How much money will she have left, if she just decides to buy 1 bottle of oil and 1 bottle of vinegar?
9. Yesterday Peter came to the store, gave the cashier 11 dollars for 3 pounds of grapes, and received some change. Today Peter came to the same store again, gave the cashier 15 dollars for 5 pounds of grapes, and also received some change. How much does the pound of grapes cost, if the change he received yesterday and today is the same?
10. Two secretaries, Barbara and Mary, need to type a 100-page document. Barbara can type it in 4 hours, Mary types slower, so it would take her 5 hours to do this. How fast can they type it together, if they divide the work between the two of them in the most efficient way?