Math 4a. Class work 26.

Revew of sets.

1. A set is a collection of well-defined objects. We can create a set just by listing all of its elements. For example, set A contains 2,5, v, n, •, ◊. We denote, $A = \{2,5, v, n, \bullet, \lozenge\}$. The second way to create a set is to describe a rule, which is applicable to all elements in the set. For example: set N is the set off all natural numbers. So we know that set B contains all natural numbers, $N = \{1, 2, 3, \dots..\}$.
2. If *B* is a set and *x* is one of the objects of *B*, this is denoted *x* ∈ *B*, and is read as "x belongs to B", or "x is an element of B". If *y* is not a member of *B* then this is written as *y* ∉ *B*, and is read as "y does not belong to B".
3. $C = \{2, 5\}$, $\quad C \subset A$

   Each element of the set C is also an element of set A, so C is subset of A.

   C is also a subset of N, since 2 and 5 are natural numbers. We can write $C \subset N$.

   The empty set is a subset of every set and every set is a subset of itself:

- $\emptyset \subset A$.
- $A \subset A$.

4. A set containing elements which are common elements of two sets is called intersection of the two sets. $C = A \cap B$.
5. Two sets can be "added" together. The *union* of *D* and *M*, denoted by *D* ∪ *M*, is the set of all things that are members of either *D* or *M*
6. We can divide set into two or more subsets in such a way that each element of the set will be in only one of these subsets, intersection of any two subsets will be an empty set. The set of non-intersecting subsets is called partition of the set. For example, the set of natural numbers N can be partitioned into two sets, of even and odd numbers. Each natural number is easer even or odd.

7. $M = \{x|\ x > 5\}$, $K = \{x|x < 20\}$

   $M \cap K$=

8. $A = \{a, b, c, d\}$, $B = \{c, d, e, f\}$, $C = \{c, e, g, k\}$.

$(A \cap B) \cap C =$

$(A \cup B) \cup C =$

9. Among 100 university students 48 are studying English, 26 are studying French, 28 are studying German, 8 are studying English and German, 8 are studying English and French, 13 are studying French and German, 24 are not studying any language. How many students are studying all three languages?

10. Set X={-5, -4, -3, -2, -1, 0, 1, 2, 3, 4, 5}

Create a set Y using the rule: $y = 2x$

Fill the table:

x											
y											

What do you see?

1. **Divisor**.

$$a:b = cRr \qquad or \qquad a = b \cdot c + r$$

Where
a is a dividend,
b is a divisor,
c is a quotient,
and r is a remainder.

Can a remainder be a negative number?

$48:7 = 6R6 \qquad or \qquad 48:7 = 7R(-1)$.?

The term “divisor” is usually used when a number is divisible by a divisor with a remainder equal to 0. How we can find all possible devisors of a given number? If this number is a prime number, then it has only two divisors, 1 and itself. If the number is not prime, then it can be represented as a product of prime numbers. Each of these prime numbers is a divisor of the number as well as a product of any combination these prime numbers. For example, let’s find all possible divisors of 36.

$$36 = 2 \cdot 2 \cdot 3 \cdot 3 = 2^2 \cdot 3^2 = (2 \cdot 3)^2$$

We can see that 2 and 3 are both divisors of 36, also 4, 9, 6, 12 and 18.

$4 = 2 \cdot 2, \quad 9 = 3 \cdot 3, \quad 6 = 2 \cdot 3, \quad 12 = 2 \cdot 2 \cdot 3, \quad 18 = 2 \cdot 3 \cdot 3$

Set of divisors of 36 contains

$$D(36) = \{2, 3, 4, 6, 12, 18, 36\}$$

How we can find common divisors of two or more numbers? The trivial answer is 1, any number is divisible by 1. If two numbers do not have common divisors except one, they are called coprime (mutually prime, relatively prime). For example, 14 and 15 are coprime numbers. but 14 and 21 are not, because they are both divisible by 7.
Can you think about other examples of coprime number?
How other common divisors of two or more numbers can be find?

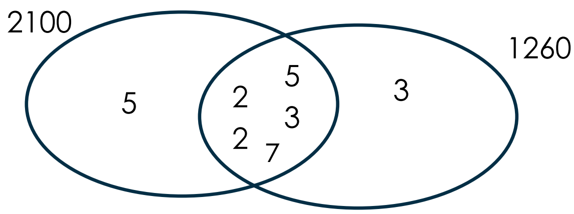
1. Prime factorization.
2. Find common prime factors, they will be common divisors.
3. Product of any combination of these prime factors will be also common divisors.

4. Product of all common prime factors will be the GCD.

Example; find GCD of
2100 and 1260.

$2100 = 2 \cdot 2 \cdot 3 \cdot 5 \cdot 5 \cdot 7$
$1260 = 2 \cdot 2 \cdot 3 \cdot 3 \cdot 5 \cdot 7$

Common prime factors will be the divisors for both numbrs ;



Both numbers are divisible by 2, 3, 5, and 7. Also they are both divisible by the product of any combination of 2, 2, 5, 3, and 7. The greatest possible number by which both 2100 and 1260 (GCD, GCF) is the product of all common prime factors.

2. **Multiple**.
Multiple of a number a is a number divisible by a. It is easy to see that any product of a and any other integer is a multiple of a. How to find the common multiple of two or more numbers? First obvious answer is their product and any of its product with another integer. For example, for number 2 and 3, common multiple will be

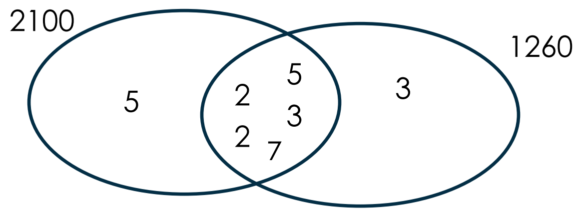
$$2 \cdot 3 = 6$$

and 12, 18, …
If two numbers are coprime numbers, the smallest common multiple will be their product. The smallest common multiple of 9 and 10 is 90.
If the numbers is not coprime:
1. Prime factorization.
2. Find the product of all elements of the union of sets of prime factors of all numbers.

$2100 = 2 \cdot 2 \cdot 3 \cdot 5 \cdot 5 \cdot 7$
$1260 = 2 \cdot 2 \cdot 3 \cdot 3 \cdot 5 \cdot 7$

$P(2100) = \{2, 2, 3, 5, 5, 7\}$
$P(1260) = \{2, 2, 3, 3, 5, 7\}$

$P(2100) \cup P(1260) = \{2, 2, 3, 5, 5, 7, 3\}$



We can see that smallest number divisible by both 2100 and 1260 is 6300

$6300 = 2 \cdot 2 \cdot 3 \cdot 5 \cdot 5 \cdot 7 \cdot 3 = 2100 \cdot 3 = 2 \cdot 2 \cdot 3 \cdot 3 \cdot 5 \cdot 7 \cdot 5 = 1260 \cdot 5$

3. Find CCD and LCM for:
   a. $a = 2^3 \cdot 3^3 \cdot 5^2,\ \ b = 2^2 \cdot 3^4 \cdot 5$
   b. 18 and 60
   c. 72, 96, 120
   d. 35, 88
4. Compute:

$$\frac{3 \cdot 3 \cdot 5 \cdot 11}{3 \cdot 11}; \qquad \frac{2 \cdot 2 \cdot 3 \cdot 5 \cdot 7}{2 \cdot 3 \cdot 7}; \qquad \frac{2 \cdot 3 \cdot 7 \cdot 13}{3 \cdot 7}; \qquad \frac{3 \cdot 5 \cdot 11 \cdot 17 \cdot 23}{3 \cdot 11 \cdot 17};$$

5. Can you represent the following numbers as the sum of prime numbers?
   10, 36, 54, 15, 27, 49