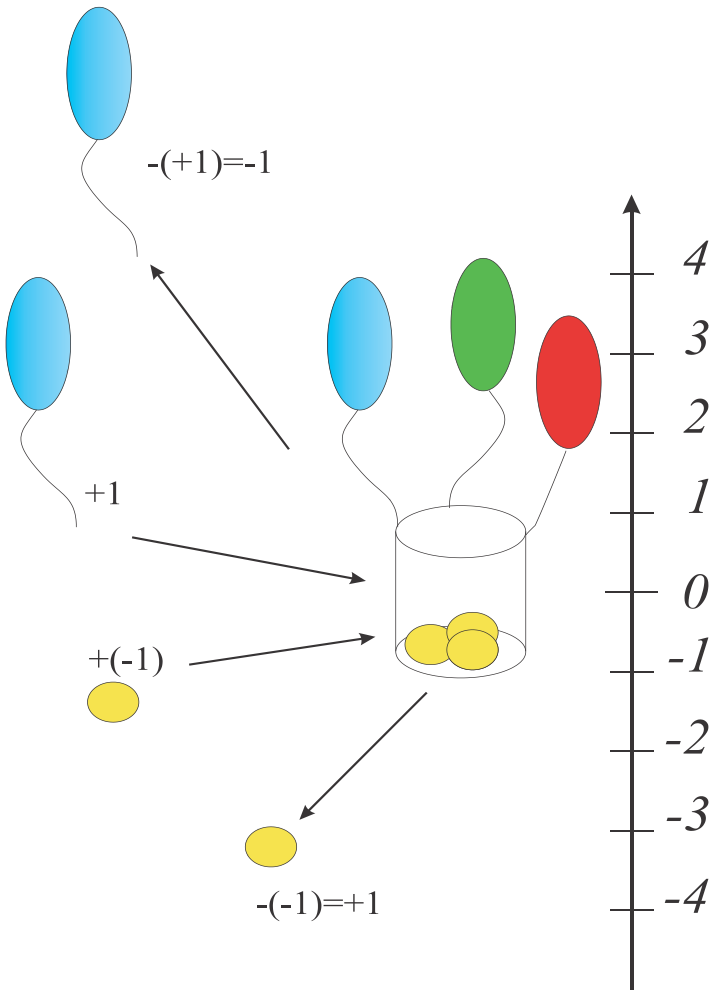
Math 4a. Class work 9.

Algebra.

Positive and negative numbers.

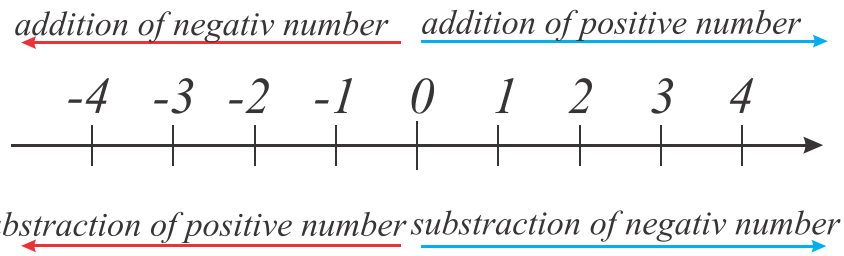
Negative numbers represent opposites. If positive represents movement to the right, negative represents movement to the left. If positive represents above sea level, then negative represents below level. If positive represents a deposit, negative represents a withdrawal. They are often used to represent the magnitude of a loss or deficiency. Negative numbers appeared for the first time in history in the Nine Chapters on the Mathematical Art, which in its present form dates from the period of the Chinese Han Dynasty (202 BC – AD 220), but may well contain much older material. Liu Hui (c. 3rd century) established rules for adding and subtracting negative numbers. By the 7th century, Indian mathematicians such as Brahmagupta were describing the use of negative numbers. Islamic mathematicians further developed the rules of subtracting and multiplying negative numbers and solved problems with negative coefficients. Western mathematicians accepted the idea of negative numbers by the 17th century. Prior to the concept of negative numbers, mathematicians such as Diophantus considered negative solutions to problems "false" and equations requiring negative solutions were described as absurd



Last year when we discuss the negative numbers we used very clear analogy of a basket with balloons and sand bags.

At the beginning basket has N balloons and N sand bags, placed at 0 position and doesn't move. Balloons represent positive numbers, sand bags represent negative numbers. If we add one balloon the basket will move one unit up. If we add one sand bag basket will move one unit down. If we remove one balloon, basket will go one unit down, which is equivalent of adding one sand bag. So $-(+1) = +(-1)$. If we remove

one sand bag, basket will go one unit up, which is equivalent of adding one balloon. So $-(-1) = +(+1)$. Let's move to number line:



Two numbers that have the same magnitude but are opposite in signs are called opposite numbers.

Exercises.

Fill up the table:

a	7	-4			5		0	
$-a$			0	-1		8		-3

1. Compare:

$-4 \quad 4$ $\qquad$ $6 \quad -4$ $\qquad$ $\frac{2}{3} \quad -\frac{3}{2}$

$-4 \quad -2$ $\qquad$ $-4 \quad 0$ $\qquad$ $-\frac{2}{3} \quad -1$

$-4 \quad -6$ $\qquad$ $-1 \quad -\frac{1}{2}$ $\qquad$ $-2 \quad \frac{1}{2}$

2. Compute:

$3 + (-2) =$ $\qquad$ $3 + (+2) =$ $\qquad$ $-3 - (-2) =$

$-3 + (-2) =$ $\qquad$ $-3 + (+2) =$

$3 - (+2) =$

$-3 - (+2) =$ $\qquad$ $-3 + (+3) =$

$3 - (-2) =$

3. Compare without calculation.

$100 - (35 - 20)$ $100 - (35 + 20)$ $100 + (35 - 20)$ $100 + (35 + 20)$

$100 - (-35 - 20)$ $100 - (-35 + 20)$ $100 + (-35 - 20)$ $100 + (-35 + 20)$

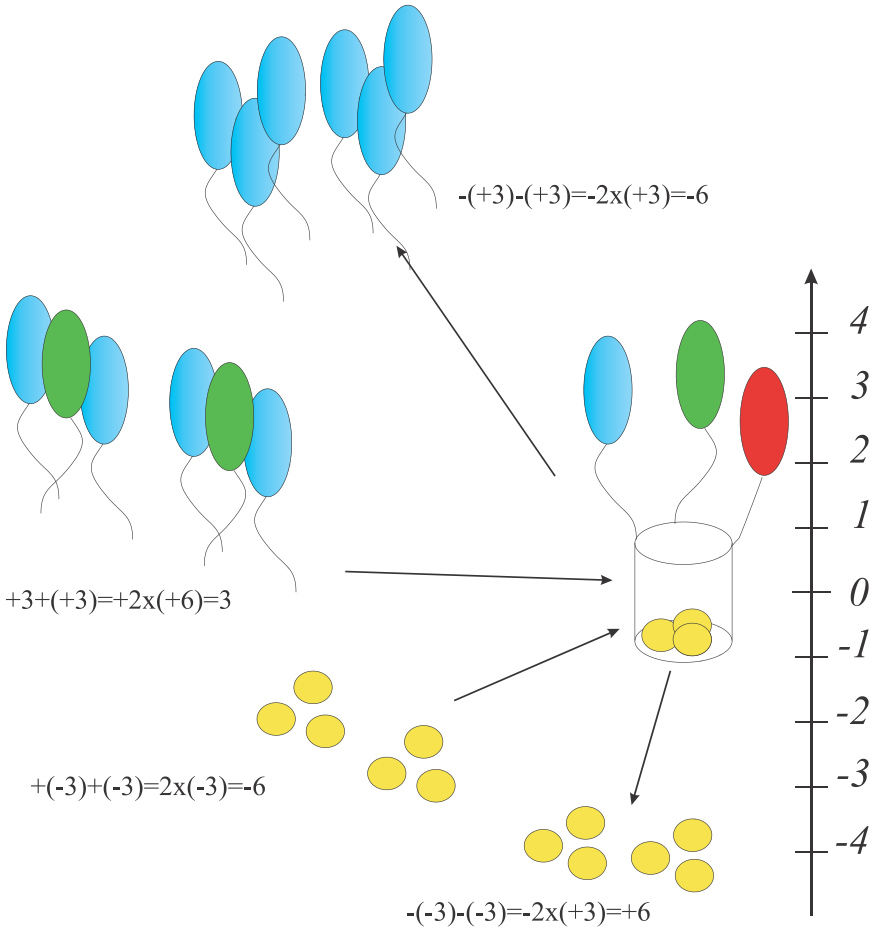
4. Rewrite without parenthesis:

$20 + (2 - 3) =$ $20 - (-2 + 3) =$

$20 - (2 - 3) =$ $20 - \big(-2 + (-3)\big) =$

Multiplication and division of negative numbers.

If we multiply 2 positive numbers we will get third positive number. What will happened if we multiply one negative and one positive number. Let's again review our analogy. In this case we will add or remove our balloons and sand bags by groups of three. Addition of to groups of 3 sand bags will drive the basket 6 units down, because we add 6 bags. So $2 \times (-3) = -6$. (We know that $-1 + (-1) + (-1) = -3$)



Removing of 2 groups of 3 sand bags will drive our basket 6 units up, which is corresponding of adding 6 balloons, so $-2 \times (-3) = 6$

Addition of 6 balloons (2 groups of three) of cause will help us to move up for 6 units. If we remove 2 groups of 3 balloons we will descend 6 units. $-2 \times (+3) = -6$.

5. Solve the following equations:

$x + 4 = -1$	$5 - x = -3$	$x - (-4) = 0$

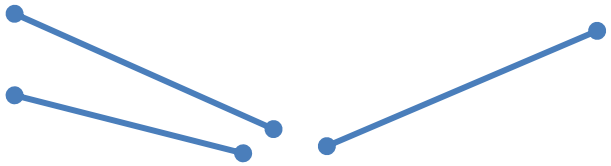
6. Positive or negative number will be the product of
   a) Two negative and one positive numbers.
   b) One negative and two positive numbers
   c) Three negative numbers.

7. A swimming pool can be filed by one pipe in 10 hours or by another pipe in 15 hours. How long it will take to fill up the pool with both pipes opened?

8. A swimming pool can be filed with one pipe in 10 hours. Full pool can be drain out with another pipe in 20 hours. How long it will take to fill up the pool with opened drain pipe?

Geometry.

Construct a triangle with sides equal to three segments:



Construct the segment, equal to 2x, 3x, where x is a given segment.