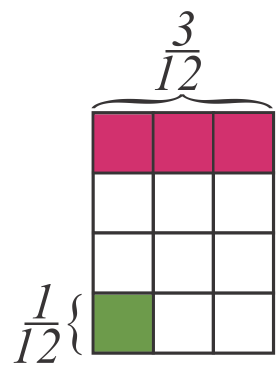
Math 4a. Class work 6.

Algebra. Fractions.

A fraction (from Latin: fractus, "broken") represents a part of a whole.

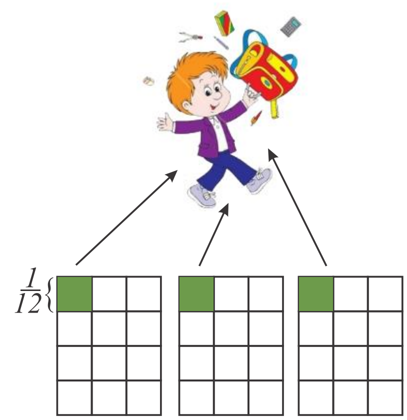
Look at the picture on the right:



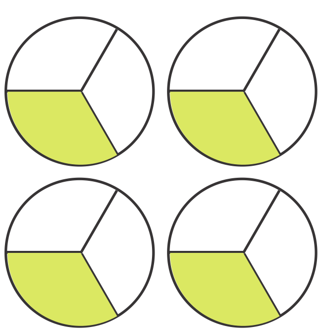
the whole chocolate bar is divided into 12 equal pieces:

$$1\ (whole\ chocolate\ bar\ ) \div 12 (equal\ parts) = \frac{1\ (whole\ chocolate\ bar\ )}{12 (equal\ parts)} = \frac{1}{12} (of\ whole\ chocolate\ bar)$$

$$\frac{1}{12} + \frac{1}{12} + \frac{1}{12} = 3 \times \frac{1}{12} = \frac{3}{12} = \frac{1}{4} = 3 \div 12$$



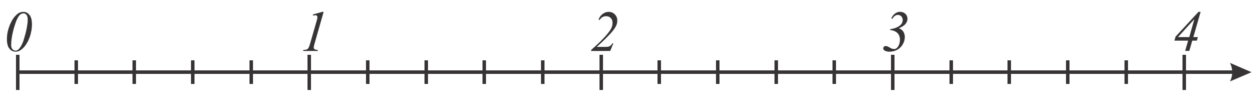
(To divide 3 chocolate bars between 12 kids we can give each kid $\frac{1}{12}$ of each chocolate bar, altogether $3 \div 12 = 3 \times \frac{1}{12} = \frac{3}{12} = \frac{1}{4}$).



To divide 4 pizzas equally between 3 friends we will give each friend $\frac{1}{3}$ of each pizza. Each friend will get $4 \div 3 = 4 \times \frac{1}{3} = \frac{4}{3}$ which is exactly 1 whole pizza $(3 \times \frac{1}{3} = \frac{3}{3} = 1)$ and $\frac{1}{3}$.

Mark following fractions on the number line:

$$\frac{1}{5}, \quad \frac{3}{5}, \quad \frac{3}{3}, \quad \frac{7}{5}, \quad \frac{10}{5}$$

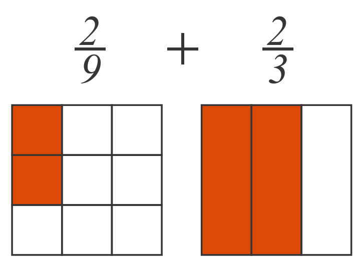


When we are talking about fraction we usually mean the part of a unit. Proper fractions are parts of a unit; improper fractions are sums of a natural number and a proper fraction. Sometimes we want to find a part of something which is not 1, but can be considered as a single object. For example, among my 30 pencils $\frac{2}{5}$ are yellow. How many yellow pencils do I have? What does it mean to find $\frac{2}{5}$ out of 30? The whole pile of all of all these pencils is a single object and we want to calculate how many pencils does a little pile of $\frac{2}{5}$ of 30 contain? $\frac{2}{5}$ is 2 times $\frac{1}{5}$, and $\frac{1}{5}$ of 30 is $30 \div 5$. So $\frac{2}{5}$ of 30 pencils will be twice more: $\frac{2}{5} \times 30 = 30 \div 5 \times 2$

Addition and subtraction of fractions with unlike denominatoers.

Let's try to add $\frac{2}{9}$ and $\frac{2}{3}$. What should we do? Why do we need to bring both fractions to the same denominator? We can add together only similar objects: apples to apples and oranges to oranges. Are two fractions $\frac{2}{9}$ and $\frac{2}{3}$ similar objects?

$\frac{2}{3} = \frac{1}{3} + \frac{1}{3}$, $\qquad \frac{2}{9} = \frac{1}{9} + \frac{1}{9}$



How we can add together

$$\frac{2}{9} + \frac{2}{3} = \frac{1}{9} + \frac{1}{9} + \frac{1}{3} + \frac{1}{3}$$

To be able to add two fractions we have to be sure that they have the same denominator. Each $\frac{1}{3}$ is exactly the same as $\frac{3}{9}$ and $\frac{2}{3} = \frac{6}{9}$

$$\frac{2}{3} \times 1 = \frac{2}{3} \times \frac{3}{3} = \frac{2 \times 3}{3 \times 3}$$

If we multiply both numerator and denominator by the same number the fraction will not change. To bring 2 fractions to the same denominators we have to multiply the numerators and the denominators of both fractions by two different numbers to get a common multiple as the denominator for both fractions. There are many common multiples of 2 numbers. Of course, one of them is their product, but is not always the simplest one. Usually, it is convenient to find LCM of these 2 (or, sometimes more than 2) numbers.

Exercises.

1. Rewrite these expression of division as fractions:
   Example: $3 \div 5 = \frac{3}{5}$

   $9 \div 5 =$ $\qquad$ $5 \div 11 =$ $\qquad$ $2 \div 6 =$

2. Compare:

   $\frac{3}{5} \quad \frac{2}{5}$ $\qquad$ $\frac{3}{5} \quad \frac{3}{8}$ $\qquad$ $\frac{3}{6} \quad \frac{1}{2}$

   $\frac{1}{5} \quad \frac{5}{1}$ $\qquad$ $\frac{4}{12} \quad \frac{3}{4}$ $\qquad$ $\frac{2}{11} \quad \frac{1}{12}$

3. Calculate:

   $\frac{1}{5} + \frac{1}{5} + \frac{1}{5} =$ $\qquad$ $\frac{2}{7} + \frac{1}{7} =$ $\qquad$ $\frac{7}{9} - \frac{3}{9} =$

4.

a. What is bigger, the number c or $\frac{2}{3}$ of the number c? Why?

b. What is bigger, the number b or $\frac{3}{2}$ of the number b? Why?

c. What is bigger, $\frac{2}{3}$ of a number m or $\frac{3}{2}$ of a number m? Why?

5. a. $\frac{1}{7}$ of all students in the class is 4. How many students are there in the class?

   b. $\frac{2}{5}$ of all students in a class is 10. How many students are there in a class?

6. In the school cafeteria there are 12 tables. There are 10 seats at each table. At the lunch time $\frac{4}{5}$ of all sits were occupied by students. How many students were in the cafeteria?

7. An apple worm was eating an apple. On the first day it ate half of the apple, on the second day it ate half of the rest, and on the third day it ate half of the rest again. On the fours day it ate all the leftovers. What part of the apple did it eat on the fourth day?

8. Peter spent 2 hours doing his homework. $\frac{1}{3}$ of this time, he spent doing his math homework and $\frac{1}{4}$ of the remaining time he spent on the history assignment. How many minutes did Peter spent on his history assignment and how many minutes did he spent doing his math homework?

9. Write the expression for the following problems:

a. 3 packages of cookies cost a dollars. How many dollars do 5 of the same packages cost?

b. 5 bottles of juice cost b dollars. How many bottles can one buy with c dollars?

10. Come up with the word problem which can be solved using the following expression:

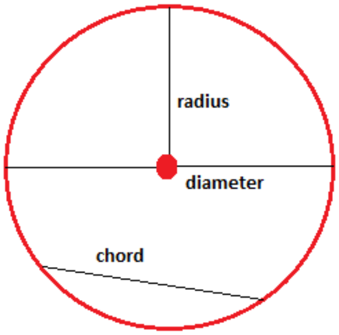
$$25 - 2 * 3;$$

(This expression is equivalent of 2 shorter expressions:

1. 2×3
2. $25 - 6$)

Geometry.

Circle is the set of all points in a plane that are at a given distance from a given point, the center.



Polygons.

In elementary geometry, a **polygon** is a plane figure that is bounded by a finite chain of straight line segments closing in a loop to form a closed chain. These segments are called its *edges* or *sides*, and the points where two edges meet are the polygon's *vertices* (singular: vertex) or *corners*. The interior of the polygon is sometimes called its *body*. An ***n*-gon** is a polygon with *n* sides; for example, a triangle is a 3-gon.