ADVANCED PHYSICS CLUB

FEBRUARY 24, 2019

Today's meeting

After reviewing the homework from last time, we discussed a derivation of ellipticity of orbits for radial force which varies like $1/r^2$.

Overview of the derivation of elliptical orbits

1. We reviewed that conservation of angular momentum implies the motion takes place in a plane.

2. We showed that when the force is $\mathbf{F} = -G_N \frac{mM}{r^2}\hat{r}$, there is a new conserved vector quantity

$$\mathbf{u} = \mathbf{v} - G_N \frac{mM}{L}\hat{\theta}\,,$$

where $\mathbf{v}$ is the velocity vector, L is the magnitude of angular momentum and $\hat{\theta}$ is a unit vector in the plane of motion, perpendicular to the position vector $\mathbf{r}$ and oriented in the direction of motion.

3. Taking the projection of the above conservation law onto the direction of $\hat{\theta}$ gives the equation for the trajectory

$$r(\theta) = \frac{R}{1 + e\cos(\theta)}\,,$$

which is an ellipse with a focus at the origin.

Homework

1. A projectile is launched from the North pole tangentially to the surface of Earth at a speed v. How long will it take for it to fall back onto the Earth's surface? Assume the Earth is a perfect sphere and neglect air resistance. The three Kepler's laws are enough to solve this problem.

2. The conserved quantity $\mathbf{u}$ from above is closely related to the so-called Laplace-Runge-Lenz vector. You can read more about it on Wikipedia: `https://en.wikipedia.org/wiki/Laplace-Runge-Lenz_vector`. How are $\mathbf{u}$ and the Laplace-Runge-Lenz vector related?

For the next meeting

The next club's meeting is at 2:40pm, room P-131, on Sunday, **March 3**.