ADVANCED PHYSICS CLUB

OCTOBER 28, 2018

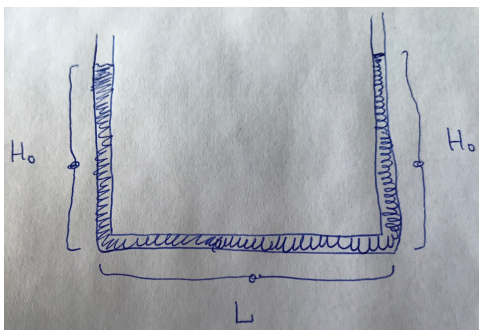
Today's meeting

Today we started with deriving trigonometric identities using Euler's formula $e^{i\alpha} = \cos\alpha + i\sin\alpha$. Then we discussed two ways of solving oscillation problems: writing the equation of motion and recognizing the energy of an oscillator (a mass at the end of a spring), $E = \frac{1}{2}m\dot{x}^2 + \frac{1}{2}kx^2$ in the total energy of the system.

Please read this document thoroughly, it will address some loose ends from class.

Discussed problems

1. Consider the system in the figure and assume that the cross section is uniformly A everywhere. What is the frequency of small oscillations about the equilibrium in the figure?



Solution: Let us denote by h the rise of the water compared to its equilibrium position in one of the tubes. We write down the total energy of the system.

$$E = E_{\text{kin}} + E_{grav}\,,$$

where because the whole fluid is moving together with speed $v = \dot{h}$

$$E_{\text{kin}} = \frac{1}{2}m_{\text{tot}}\dot{h}^2\,.$$

The gravitational energy is trickier to determine. The error proof way would be to determine the change in the position of the center of mass of the system, and use $E_{grav} = m_{\text{tot}}g\Delta z_{\text{CM}}$. Instead, we can argue as follows: to get to the out of equilibrium configuration we have to move a piece of fluid of mass ρAh from one tube to the other, and lift it up by h, giving

$$E_{\text{grav}} = \rho Agh^2\,.$$

Hence the total energy is

$$E = \frac{1}{2}\rho A(2H_0 + L)\dot{h}^2 + \rho Agh^2\,.$$

We recognize the energy of an oscillator ($E = \frac{1}{2}m\dot{x}^2 + \frac{1}{2}kx^2$) in this expression with the identification $m = \rho A(2H_0 + L)$ and $k = 2\rho Ag$, thus the frequency of oscillations is

$$\omega^2 = \frac{2\rho Ag}{\rho A(2H_0 + L)} = \frac{2g}{(2H_0 + L)}\,.$$

In class we got the same result, but there was a bit of confusion about how we got there. David's original solution was to say that the center of mass accelerates as $\ddot{h}$, and that the only force that the net forces that acts is $2\rho Agh$, hence

$$\rho A(2H_0 + L)\ddot{h} = -2\rho Agh\,, \tag{1}$$

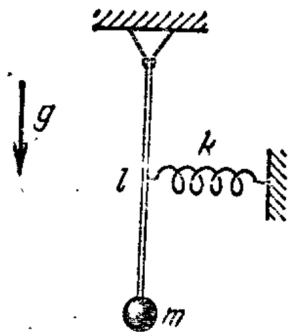
giving the same frequency as above. It was unclear to us how to argue for the net force being $2\rho Agh$, so we tried to decompose the fluid into two parts, the $2h$ part in one tube, and the rest of the fluid

of length $2H_0 + L - 2h$. It is true that this latter part of the fluid is pushed down by the weight of the $2h$ part, but **because the mass of this long part is changing in time, $F \neq ma$ in such a situation.** I failed to point this out in class. In the case of mass changing in time, we have to use a generalization:

$$F = \frac{dp}{dt} = m(t)a + \dot{m}(t)v\,,$$

where $p = mv$ is the momentum. Can you recover (1) from combining this equation with the line of reasoning sketched above?

2. We solved the pendulum problem using the equation of motion method, and then considered the system on the figure below.



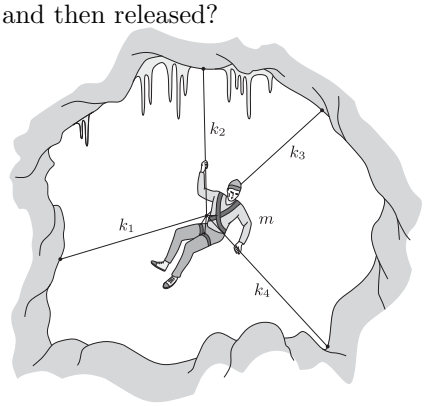
We determined the frequency of oscillations of this system using the energy method to be $\omega^2 = \frac{g}{\ell} + \frac{k}{4m}$.

Homework

1. Consider problem **2.** above.

- Try to solve the problem using the equation of motion method by including only the most obvious forces that act? Did you get $\omega^2 = \frac{g}{\ell} + \frac{k}{2m}$? What are the non-obvious forces?
- In the motion of rigid bodies its often easier to work with notions like angular momentum, torque, etc. If you know these concepts, can you solve the problem with these?

2. A mountaineer (a former circus artist) has to spend the night on the (vertical) side of a high mountain. So, as shown in the figure, he clamps himself to four carabiners fixed to the rock face using four extraordinarily flexible springs. The masses of the springs and their unstretched lengths are negligible and the mountaineer can be considered – for the sake of simplicity – as a point-like body with a mass m. What is the mountaineer's period of oscillation if he is displaced from his equilibrium position and then released?



Hint: Describe, in vector notation, the equation determining the mountaineer?s equilibrium position and the equation of motion when his body is displaced from equilibrium.