## PROBABILITIES AND GEOMETRY

JANUARY 6, 2019

1. Two peple have agreed to meet at certain location between $1 \mathrm{pm}-2 \mathrm{pm}$. Each person arrives at the location at a random mment between 1 and 2 ; if the other person is not there, he waits for 15 minutes and then leaves.

What is the probability that they will meet?
2. To get to school, Alex has to take the subway northbound. Every day he comes to the subway station at some random time between 7 am and 7:20 am to take the train. However, he decided that if the southbound train comes first, he would take it and go to the park instead.

At the end of the year, he found out that he had gone to the park twice as frequently as he would go to school.

How is it possible if the frequency of northboud and southbound trains is the same - each of them runs exactly every 5 minutes?
3. Two numbers are chosen randomly in the interval $[0,1]$. What is the probability that their sum is larger than $1 / 2$ ?
4. Three points are chosen at random on the circle. What is the probability that the triangle formed by these three points is obtuse?
5. Three points $A, B, C$ are chosen at random in the interval $[0,1]$.
(a) What is the probability that $A$ is the largest?
(b) What is the probabiity that $A<B<C$ ?
6. (a) Three points are chosen at random on the circle. What is the probability that the triangle formed by these three points contains the center of the circle?
*(b) Four points are chose at random on the surface of the sphere. What is the probability that the tetrahedron formed by these four points contains the center of the circle?
7. Four points $A, B, C, D$ are chosen on the circle at random. What is the probability that chords $A B$ and $C D$ intersect?
*8. Buffon's needle.
An infinite sheet of paper is lined horizontaly, with lines 1 inch apart. We are dropping a 1 inch long needle on the paper at random.
(a) Show that if we know that the needle fell so that it forms angle $\alpha$ (where alpha is measured in radians, $0<\alpha<\pi / 2$ ) with the horizontal, then the probability that the needle crosses one of the lines is equal to $\sin \alpha$.
(b) Deduce from this that the probability that the needle will cross one of the lines is equal to the average value of $\sin \alpha$ over the interval [ $0, \pi / 2$ ], or in other words

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\frac{1}{\pi / 2} \int_{0}^{\pi / 2} \sin \alpha d \alpha
$$

(c) FInd this probability.
9.
10.

