

Homework 19.

Magnetic flux and the Faraday's law of induction.

Last class we discussed *magnetic flux*. Let us consider a flat (for simplicity) wire loop of an arbitrary shape. Let us place this loop in a spatially uniform magnetic field so that magnetic induction vector \mathbf{B} and the line perpendicular to the wire plane form angle α .

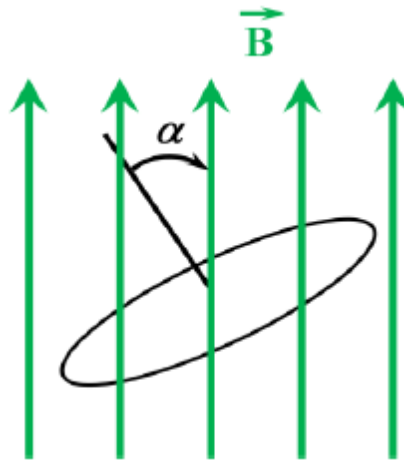


Figure 1.

In this case, magnetic flux Φ through the loop can be calculated as:

$$\Phi = B \cdot S \cdot \cos\alpha \quad (1)$$

Here B is the magnetic induction; S is the area of the wire loop. Magnetic flux is a scalar. One can see from the formula (1) that there are 3 parameters which determine the magnetic flux through the loop:

1. Magnetic field B .
2. Loop's area S .
3. Angle α between the magnetic field and the direction perpendicular to the loop's plane.

Experiment shows that change of the magnetic flux through the closed wire loop leads to onset of electrical current in the loop. The magnitude of the current depends on the loop's resistance and the rate of the magnetic flux change. Changing the magnetic flux through the loop are somehow equivalent to inserting a battery with a certain e.m.f. E (this is electromotive force – “voltage” of the battery). The faster we are changing the flux, the higher E . So,

$$|E| = \left| \frac{\Delta\Phi}{\Delta t} \right| \quad (2)$$

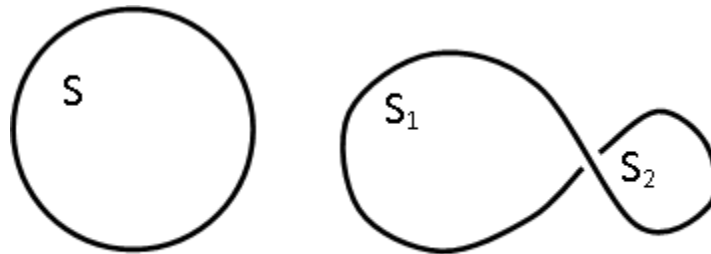
Here $\Delta\Phi$ is a change of the magnetic flux during the time Δt . The equation (2) is called Faraday's law of induction. The experiment demonstrating the induction phenomenon was performed by

Michael Faraday in 1831. The direction of the induced current depends on the nature of the magnetic flux change (increasing or decreasing).

Magnetic flux is measured in webers (W). $1W=1 \text{ Tesla} \times 1 \text{ meter}^2$. The unit is named after a German physicist Wilhelm Eduard Weber (1804-1891).

Problems:

1. Uniform magnetic field in vacuum changes at a rate of $0.1T/s$. A $0.25m^2$ wire loop is placed in the field, the angle between perpendicular to the loop plane and the field direction is 60° . Find EMF (electromotive force - voltage) induced in the loop.
2. Magnetic flux through the wire loop changes uniformly from 0.0002 W (Weber= $\text{Tesla} \times m^2$) to $0.0005W$ for $10s$. The resistance of the wire is 3Ω . Calculate the current flowing through the wire as the flux is changing.
3. Two wire loops shown in the picture below are in uniform magnetic field which is perpendicular to the plane of the loops. As the magnetic field changes, electromotive force induced in the left loop is E . Find EMF induced in the right loop (assume that you know S , S_1 , S_2)



4. (*This is an optional problem*) A rectangular wire frame with a side L is placed into uniform magnetic field with magnetic induction B directed perpendicularly to the frame plane (“to us”). There is an additional piece of wire AB which connects two long sides of the frame and can slide without breaking electrical contact. The resistance of the wire is R (the resistance of the frame itself is 0). You are moving the wire at a speed V along the frame as it is shown in the picture. Find magnitude of the current in the wire AB .

